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Fixed-point fast sweeping weighted essentially non-oscillatory method for multi-commodity continuum traffic equilibrium assignment problem

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ABSTRACT

This work presents a fixed-point fast sweeping weighted essentially non-oscillatory method for the multi-commodity continuum traffic equilibrium assignment problem with elastic travel demand. The commuters' origins (i.e. home locations) are continuously dispersed over the whole city with several highly compact central business districts. The traffic flows from origins to the same central business district are considered as one commodity. The continuum traffic equilibrium assignment model is formulated as a static conservation law equation coupled with an Eikonal equation for each commodity. To solve the model, a pseudo-time-marching approach and a third order finite volume weighted essentially non-oscillatory scheme with Lax–Friedrichs flux splitting are adopted to solve the conservation law equation, coupled with a third order fast sweeping numerical method for the Eikonal equation on rectangular grids. A fixed-point fast sweeping method that utilizes Gauss–Seidel iterations and alternating sweeping strategy is designed to improve the convergence for steady state computations of the problem. A numerical example is given to show the feasibility of the model and the effectiveness of the solution algorithm.

1. Introduction

Continuum modeling of traffic equilibrium problems in a transportation system provides an efficient way to deal with large-scale dense road networks [1]. In the continuum modeling approach, a dense road network can be approximated as a two-dimensional (2D) continuum in which road users can choose their routes freely. This approach can reduce problem size for a dense road network in a large area. Here, the problem size depends on the method used to approximate the modeling region rather than on the actual number of nodes and links modeled. The fundamental assumption made is that differences in the modeling characteristics of adjacent areas within a network (e.g. travel costs and demand patterns) are relatively small in comparison with the variations over the entire network. Therefore, the characteristics of the network, such as traffic demand, flow intensity, density and travel cost, can be represented by smooth mathematical functions in a continuum model [2]. The relationships among these macroscopic variables can be well established with less data. In

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view of these advantages, the continuum modeling approach is now widely used to model the multi-commodity cost-flow relationship [3,4], market share determination [5,6], cordon-based congestion pricing problems [7], housing problems [8,9], dynamic traffic assignment problems [1,10] and pedestrian flow problems [11–18].

For continuum transportation problems, flow variables and trip demand satisfy the flow conservation condition in a 2D continuum [3]. Under the static assumption of the demand and supply sides of a transportation system, the flow conservation condition is involved with a stationary problem of hyperbolic conservation laws with source term. Suppose that, every road user in a transportation system has perfect traffic information and chooses an optimal route that minimizes either the total travel time or cost to his/her travel destination. A user equilibrium condition is reached based on this user-optimal route choice behavior of road users [3,7]. The continuum traffic equilibrium model composed of the flow conservation condition and the user equilibrium condition can be effectively solved by mathematical tools, such as the finite element method [3,6,19]. However, the finite element method results in a large system of algebraic equations and thus is subject to certain numerical difficulties in saving computing time and computer memory space, especially if the accuracy of the algorithm is improved or the mesh of the computational domain is refined.

To deal with nonlinear stationary conservation laws, a typical approach is to use an explicit time-stepping or pseudo-time-stepping technique [20–22]. This technique adds a pseudo-time variable which turns a stationary problem into a time dependent one and evolves the solution to steady state. However, the computational efficiency of this approach is restricted by a Courant–Friedrichs–Lewy (CFL) condition for the discrete time step size and it takes significant time to evolve the solution of the pseudo-time problem to steady state [23]. Recently, a fast sweeping method, which was originally designed for solving static Hamilton–Jacobi equations [24–27], has been applied to solve hyperbolic conservation laws with source terms [23,28]. This method is an efficient Gauss–Seidel iterative numerical scheme to the boundary value problems with solution information propagating along characteristics starting from the boundary. The resulting algorithm typically has linear computational complexity. In [27], Zhang et al. adopted the Gauss–Seidel idea and alternating sweeping strategy to the time-marching type fixed-point iterations to solve static Hamilton–Jacobi equations. This fixed-point fast sweeping approach has an explicit form and does not involve inverse operation of nonlinear local systems. Hence, it can be utilized to solve very general stationary problems of hyperbolic conservation laws using any monotone numerical fluxes and high order approximations, e.g. weighted essentially non-oscillatory (WENO) approximations [29,30].

In this paper, we extend the fixed-point fast sweeping WENO method to solve the multi-commodity continuum traffic equilibrium assignment problem. The commuters' origins (i.e. home locations) are assumed to be continuously dispersed over the whole city with several highly compact central business districts (CBDs). The trip demand to each CBD (i.e. destination) is elastic and is considered to be dependent on the total travel cost to that CBD. The traffic flows from origins to the same destination are considered as one commodity. The traffic equilibrium assignment problem is formulated as a static conservation law equation coupled with an Eikonal equation for each commodity. The fixed-point fast sweeping method incorporated into a third order finite volume WENO scheme with Lax–Friedrichs flux splitting [29] is used to solve the conservation law equation, coupled with a third order fast sweeping numerical method for the Eikonal equation on rectangular grids. We also give a numerical example to test the rationality of the model and the effectiveness of the solution algorithm.

The rest of the paper is organized as follows: in Section 2, the mathematical formulation for the multi-commodity continuum traffic equilibrium assignment problem is described in detail. Section 3 gives a numerical algorithm for the continuum equilibrium model of multi-commodity traffic flows. The numerical results are presented in Section 4. Finally, some concluding remarks are given in Section 5.

2. Problem formulation

Consider a city with several highly compact CBDs, as shown in Fig. 1. The city with a very dense transportation network can be approximated as a 2D continuum [3]. The commuters' origins (i.e. home locations) are continuously distributed outside the CBDs. It is assumed that commuters travel from origins to their chosen CBD along the least costly route in the morning, and vice versa in the evening. The traffic flows from origins to the same CBD are considered as one commodity. Denote the region of the city as Ω, the boundary of the city as $\partial\Omega$, and the boundary of the CBD for commodity m as Γ_m, $m = 1, 2, \ldots, M$, where M is the number of commodities in the city.

The demand distribution of commodity m is represented by $q_m(x, y)$, where $q_m(x, y)dxdy$ is the travel demand generated from location (x, y) to CBD m. To consider the elasticity of travel demand, $q_m(x, y)$ is assumed to be associated with the total travel cost of commodity m from location (x, y) to the CBD [3,4], and can be written as

$$q_m(x, y) = D_m(\phi_m(x, y)),\ (x, y) \in \Omega. \tag{1}$$

Here, D_m is a monotonically decreasing function and $\phi_m(x, y)$ is the total travel cost incurred by commodity m traveling from location (x, y) to destination Γ_m where $\phi_m(x, y) = 0$.

Let $f_m(x, y)$ is the flow intensity (or norm) of the flow vector $\mathbf{f}_m = (f_{xm}, f_{ym})$ for commodity m, i.e. $f_m = \sqrt{f_{xm}^2 + f_{ym}^2}$. Here, f_{xm} and f_{ym} are the flow flux of commodity m in the x and y directions, respectively. For each commodity, the flow vector $\mathbf{f}_m$ and trip demand $q_m(x, y)$ must satisfy the flow conservation condition inside the domain of the city as

$$\nabla \cdot \mathbf{f}_m(x, y) = q_m(x, y),\ (x, y) \in \Omega. \tag{2}$$