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An analytical approach for postbuckling of eccentrically or concentrically stiffened composite double curved panel on nonlinear elastic foundation

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ABSTRACT

This paper presents a detailed analytical investigation on the buckling and postbuckling behavior of laminated composite double curved panels with eccentrically and/or concentrically ortho-grid stiffeners subjected to in-plane compression, lateral pressure, thermal environment, and combined loads. The panels are surrounded by three parameter elastic foundations. Different types of simple-supported boundary conditions are considered. The equilibrium and compatibility equations of panel are derived based on Kirchhoff assumptions incorporating nonlinear von-Karman relations. The stress function and Galerkin method are applied to obtain explicit expressions of the buckling load and load-deflection relations. New results are presented to show effects of the combined loads, position of stiffener, and elastic foundation. As a key finding of these results, the buckling load and the postbuckling curves of concentrically stiffened panels are higher than eccentrically stiffened panels.

1. Introduction

Stiffened laminated composite materials truly have a remarkable usage in airplane structures such as fuselage, wing, and nose cone. During the operational life, these structures are likely subjected to the combinations of aerodynamic and environmental loads such as lateral pressure, in-plane compression, and temperature rise. Therefore, the analysis of stability in buckling range and even postbuckling of such structures is one of the important researches and is of interest in design.

For limited studies of laminated composite panels, Huang and Tauchert [1] established the large-deflection equations for laminated composite double curved panels and treated the equilibrium paths of panels subjected to thermal loads by finite element method. Chang and Librescu [2] reported a series of investigations on the postbuckling behavior of composite double curved panels under uniaxial/biaxial compressive edge loads and lateral pressure field. Singha et al. [3] studied the thermomechanical postbuckling analysis of laminated composite double curved panels based on a first order shear deformation theory. In order to improve the postbuckling strength of laminated composite materials, Shen [4,5] presented the postbuckling behavior of a hybrid laminated panel with piezoelectric layers subjected to thermomechanical loads.

In addition, a sizable literature has developed on the buckling and postbuckling of one and/or double curved panels which made of functionally graded materials [6–16]. Shen and Liew [6] employed a singular perturbation technique to

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Nomenclature

x, y, z	cartesian coordinate
$a,\ b,\ h$	length of longitudinal and transversal edges and total thickness of the panel
$R_x,\ R_y$	radii of longitudinal and transversal curvatures
$d_{11s},\ d_{22s}$	width of transversal and longitudinal ribs
$S_{11s},\ S_{22s}$	distance between two parallel transversal and longitudinal ribs
$h_s,\ 2h_c$	thickness of stiffener and laminated composite
$k_0, k_1, k_2,\ K_0^*, K_1^*, K_2^*$	parameters of foundation such as Winkler, Pasternak, and softening/hardening and dimensionless forms of them
$\sigma_{xx},\ \sigma_{yy},\ \tau_{xy}$	normal stresses in x and y directions and shear stress in plane xy
$\varepsilon_{xx},\ \varepsilon_{yy},\ \gamma_{xy}$	normal strains in x and y directions and shear strain in plane xy
$\varepsilon_{xx}{}^{(0)},\ \varepsilon_{yy}{}^{(0)},\ \gamma_{xy}{}^{(0)}$	strain components of middle surface of the panel
$\varepsilon_{xx}{}^{(1)},\ \varepsilon_{yy}{}^{(1)},\ \gamma_{xy}{}^{(1)}$	curvatures
$u_0,\ v_0,\ w_0$	displacement components of middle surface of the panel
k	number of layers in laminated composite
α	fiber orientation respect to the longitudinal edge
$\alpha_x,\ \alpha_y,\ \alpha_{xy}$	thermal expansion coefficients of a ply
$\Theta,\ \Delta T$	thermal load and uniform temperature rise
$\bar{Q}_{ij}(i, j = 1, 2, 6)$	stiffness matrix of a ply
$Q_{ij}{}^{s}\ (i, j = 1, 2, 6)$	stiffness matrix of stiffener
E_s	elastic modulus of stiffener
$N_x, N_y, N_{xy}, M_x, M_y, M_{xy}$	force and moment resultants
$N_x{}^T, N_y{}^T, N_{xy}{}^T, M_x{}^T, M_y{}^T, M_{xy}{}^T$	thermal force and moment resultants
$A_{ij}, B_{ij}, D_{ij}\ (i, j = 1, 2, 6)$	in-plane, coupled, and bending stiffness of laminated composite
$I_i\ (i = 1, 2, 3)$	inertia moments of stiffener
F, C_1, C_2, C_3	stress function and its coefficients
$AQ_{ij}\ (i, j = 1, 2, 6)$	inverse of total in-plane stiffness
N_{x0}, N_{y0}, η	pre-buckling forces per unit length and their ratio
F_x, F_y	compressions at longitudinal and transversal edges
q	uniform lateral pressure
W	maximum deflection
$m,\ n,\ \overline{m},\ \overline{n},\ \overline{m}_o,\ \overline{n}_o$	half waves of longitudinal and transversal directions
$\beta_i\ (i = 1, 2, ..., 6)$	functions in terms of $AQ_{ij}\ (i, j = 1, 2, 6)$ and coupled stiffness
$AQ_x{}^T, AQ_y{}^T$	thermal stiffness of laminated composite

study postbuckling of functionally graded cylindrical panels with piezoelectric actuators under thermal loads. Yang et al. [7] investigated the thermomechanical buckling and postbuckling of functionally graded cylindrical thin panels by using the classical theory. Woo et al. [8] presented an analytical solution for postbuckling of moderately thick functionally graded plates and shallow cylindrical shells under edge compression and thermal loadings. Duc and Van Tung [9,10] and Duc and Quan [11,12] derived closed form solutions of buckling and postbuckling of FGM cylindrical panels and double curved panels, respectively. Tung [13] investigated the effects of tangential edge constraints and elastic foundations on postbuckling behavior of functionally graded cylindrical panels. Van Tung and Duc [14] using a higher order shear deformation theory analyzed nonlinear response and postbuckling of FGM double curved panels under different thermal and mechanical loads. Thang et al. [15] investigated the effects of variable thickness and imperfection on nonlinear buckling of FGM cylindrical panels. Kar and Panda [16] employed the nonlinear finite element method to obtain postbuckling of FGM double curved panels subjected to edge compressions. Recently, Duc et al. [17–19] using an eccentrically stiffener in inner surface of FGM panels presented the improved postbuckling paths of cylindrical and double curved panels.

Due to complexity of constitutive equations, only a few researchers have attempted to extend formulations of stiffened laminated composite panels and the most of them are experimental reports. Feng et al. [20] presented an experimental set up for impact damage, buckling, and postbuckling of stiffened composite flat panels under axial compressions. Zhang et al. [21] tested stiffened composite panel under mechanical and hygrothermal loads to drive buckling and failure loads.

In recent years, the cylindrical panels and shells surrounded by the elastic foundation began to be investigated in a variety of industrial applications. In this context, Tornabene [22] and Tornabene et al. [23] studied free vibration, static and dynamic analysis of composite double curved panels and shells on two parameter elastic foundations named as Winkler and Pasternak. Sofiyev et al. [24] investigated the buckling load of FGM truncated conical shells on two parameter elastic foundations. The nonlinear effect of the elastic foundation on the static analysis of composite double curved panels was presented by Tornabene et al. [25]. Zhang and Zhou [26] imported the nonlinear elastic foundation into postbuckling behavior of rectangular plates.