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Computation of the configuration degree of freedom of a spatial parallel mechanism by using reciprocal screw theory

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Abstract

In this paper, we propose a programmable algorithm to investigate the *configuration degree of freedom* (CDOF) of a spatial parallel mechanism in one Cartesian coordinate system with reciprocal screw theory. According to the physical meaning of reciprocal screws, we first obtain the terminal constraints of every kinematic chain which connects the end-effector with the fixed base, and then gain the free motion(s) of the end-effector and its *degree of freedom* (DOF). Through analyzing the controllability of the end-effector, we investigate the instantaneous screw expressions for the free motions of the end-effector of a spatial parallel mechanism and the CDOF of the mechanism. The whole mathematical analysis process can be directly embedded in a kind of CAD software, in which the imperfect Kutzbach Grübler formulas or its amendments are mostly utilized to analyze the DOF.

Keywords: Configuration degree of freedom; End-effector; Spatial parallel mechanism; Terminal constraints; Reciprocal screw theory

1. Introduction

Determining the independent motions of an end-effector is the key problem of mechanism analysis, while realizing the prescribed motion(s) through an end-effector is the focus of mechanism synthesis. Consequently, DOF of a mechanism is the primary problem to be solved both in analysis and in synthesis of mechanisms. However, the currently received formulas of DOF are those of Kutzbach Grübler's [1,2], which have been found not to work for a certain mechanisms. In theories and applications, many scholars [2–5] have brought forward kinds of amendments for Kutzbach Grübler formulas, which are often divided into two types, one is for planar and spherical mechanisms and the other is for spatial mechanisms, but the essence and geneses of them are consistent.

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There is the following Kutzbach Grübler's formula [6,7] for planar and spherical mechanisms:

$$M = 3(n - j - 1) + \sum_{i=1}^{j} f_i \tag{1}$$

There is the following Kutzbach Grübler formula [6,7] for spatial mechanisms:

$$M = 6(n - j - 1) + \sum_{i=1}^{j} f_i \tag{2}$$

where n represents the number of the total members of the system, j represents the number of the total kinematic pairs, f_i represents the number of DOF of the ith kinematic pair.

Waldron [2] and Hunt [3, pp. 34–35, p. 376, p. 382] proposed the following formula:

$$M = d(n - g - 1) + \sum_{i=1}^{g} f_i \tag{3}$$

where d represents the order of the mechanism, and $d = 6 - \lambda$, λ represents the number of the common constraints of a mechanism, n represents the total number of the members in the mechanism, g represents the total number of the kinematic pairs of the mechanism, f_i represents the DOF of the ith kinematic pair, l represents the number of the independent loops of a mechanism.

Hunt also gave a more general formula in his treatise [3]:

$$M = \sum_{i=1}^{g} f_i - \sum_{j=1}^{l} d_j \tag{4}$$

where d_j ($j = 1, \ldots, l$) is the order of the jth ($j = 1, \ldots, l$) loop.

As to a history of the calculation of the mobility of a mechanism, Gogu [8] made a relatively good historical review. Details about the various methods presented in the literatures during the past 150 years can be found in [8].

In [6,7], we have already analyzed the restrictions of the above methods from different aspects, and presented analytical methods about the DOF of an end-effector and the CDOF of a mechanism. In this paper, we will concentrate on the computation of CDOF of a spatial parallel mechanism in a fixed (absolute) Cartesian coordinate system with reciprocal screw theory [3–5,9–11]. In brief review the steps addressed in [6,7], we first analyze the terminal constraints of each kinematic chain of the end-effector, and then investigate its free motion(s) at a fixed Cartesian coordinate system. The free motion(s) of the end-effector and the constraints exerted to it by the kinematic chains are all expressed in Plücker coordinates at the same Cartesian coordinate system.

2. Primary theory of reciprocal screws and the analytical degree of freedom method of terminal constraints of an end-effector

According to reciprocal screw theory [2–5,9–11], a screw \$ is defined by a straight line with an associated pitch h and is conveniently denoted by six Plücker homogeneous coordinates:

$$\$ = (\mathbf{s} \quad \mathbf{s}_0 + h\mathbf{s}) \tag{5}$$

where $\mathbf{s}$ denotes direction ratios pointing along the screw axis, $\mathbf{s}_0 = \mathbf{r} \times \mathbf{s}$ defines the moment of the screw axis about the origin of the coordinate system, $\mathbf{r}$ is the position vector of any point on the screw axis with respect to the coordinate system. Consequently, the screw axis can be denoted by the Plücker homogeneous coordinates $\$_{\text{axis}} = (\mathbf{s} \quad \mathbf{s}_0)$.

Assume

$$\begin{cases} \mathbf{s} = (L \quad M \quad N), \\ \mathbf{s}_0 + h\mathbf{s} = (P \quad Q \quad R) \end{cases} \tag{6}$$