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Excess pore water pressure due to ground surface erosion

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ABSTRACT

The Laplace transform is applied to solve the groundwater flow equation with a boundary that is initially fixed but that starts to move at a constant rate after some fixed time. This problem arises in the study of pore water pressures due to erosional unloading where the aquifer lies underneath an unsaturated zone. We derive an analytic solution and examine the predicted pressure profiles and boundary fluxes. We calculate the negative pore water pressure in the aquifer induced by the initial erosion of the unsaturated zone and subsequent erosion of the aquifer.

1. Introduction

Erosional unloading is the process whereby surface rocks and soil are removed by external processes, resulting in changes to water pressure within the underlying aquifer [1]. Jiao and Zheng [2] used vertical one-dimensional numerical models to investigate abnormal fluid pressures in geologic formations caused by gravitational loading or unloading due to deposition or erosion in sedimentary basins.

We consider a mathematical model of changes in excess pore water pressure as a result of erosional unloading. An equivalent porous medium description is used to model the resulting flow [3]. This approach has been shown to be a good model of flow in aquifers [4]. Neuzil and Pollock [5] studied this process in the case where the water table initially coincides with the surface. We generalize this case to an ideal aquifer which is initially separated from the ground surface by an unsaturated zone. Rates of erosion are discussed in [6,7], but in terms of representative values and without addressing temporal variability. In the absence of further information, we consider steady erosion here as a first step.

The problem is solved using the Laplace transform in conjunction with the boost operator derived by King [8]. The boost operator is used to boost the solution in the Laplace domain into a frame of reference moving at constant velocity with respect to the original frame. This allows one to solve the the erosional unloading problem in which one boundary moves.

We use our solution to analyze the evolution of the pressure during erosion of the aquifer for small and large erosion rates. We examine the flux at the boundaries a function of time and derive a quasi-steady approximation valid for very small erosion rates in the appendix.

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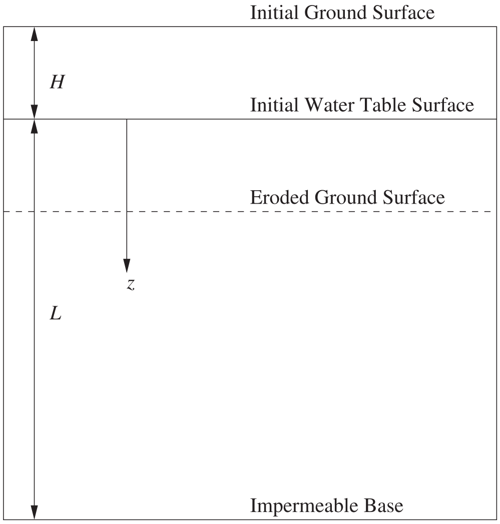

Fig. 1. System configuration.

2. Problem formulation

The model studied by Neuzil and Pollock [5] consists of a single layer of saturated aquifer where the water table is near the surface. This layer is bounded at the bottom by an impermeable layer.

Our model does not assume that the water table is next to the surface; instead we take the unsaturated zone to have non-negligible thickness (see Fig. 1). The capillary and soilwater zones are taken to have negligible thickness and are not considered. The underlying layer below is taken to be impermeable [1]. While both the permeable and impermeable cases are mentioned in [5] and both can be treated using the present approach, the latter is more relevant to applications and is hence considered here. Neuzil and Pollock [5] analyzed the following inhomogeneous equation for groundwater flow:

$$c\frac{\partial^2 p'}{\partial z^2} = \frac{\partial p'}{\partial t} - \rho_s g\frac{\partial l}{\partial t}. \tag{1}$$

This equation comes from Darcy's Law and conservation of mass applied to volume elements within the aquifer. Our source term differs from that of Neuzil and Pollock in the time interval before erosion and in the time interval during erosion. The rate of erosion $\partial l/\partial t = b$ will be assumed to be constant, and the aquifer is homogeneous.

Let the unsaturated zone and the aquifer have initial thicknesses of H and L, respectively. It follows that the permeable layer is at an initial depth of $H + L$ from the ground surface. The coordinate system is chosen so that the origin coincides with the initial depth of the water table. The z-coordinate will be taken to point down (see Fig. 1).

2.1. Governing equations

For the period before erosion, the governing equation is

$$\frac{\partial p}{\partial t} - c\frac{\partial^2 p}{\partial z^2} = -\gamma\rho_m gb. \tag{2}$$

Here, p is the excess pore water pressure, $c = K/S$ with hydraulic conductivity K and specific storage S, ρ_m is the moist density of the unsaturated zone, γ is the loading efficiency and g is gravity.

The initial condition is $p = 0$ at $t = 0$, while the boundary conditions are $p = 0$ at $z = 0$ and $\partial p/\partial z = 0$ at $z = L$. Erosion starts at $t = H/b$, and during erosion the field equation is

$$\frac{\partial p}{\partial t} - c\frac{\partial^2 p}{\partial z^2} = -\gamma(\rho_s - \rho_f)gb. \tag{3}$$

Here, ρ_s is the saturated density of the aquifer and ρ_f is the groundwater density. The boundary condition at the bottom of the aquifer remains $p = 0$, but now the upper boundary moves, so that $p = 0$ at $z = bt - H$.

We non-dimensionalize using L for length, $c^{-1}L^2$ for time and $\gamma\rho_m gbc^{-1}L^2$ for pressure. The non-dimensional equations are then

$$\frac{\partial p}{\partial t} - \frac{\partial^2 p}{\partial z^2} = \begin{cases} -1 & \text{for } t < t_0, \\ -r & \text{for } t > t_0, \end{cases} \tag{4}$$