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Singular perturbed vector field method applied to combustion in diesel engine: Continuous case with thermal runaway

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ABSTRACT

In this study, we employ the well-known method of a singularly perturbed vector field (SPVF) and its application to the thermal runaway of diesel spray combustion. Given a system of governing equations, consisting of hidden multi-scale variables, the SPVF method transfers and decomposes such a system into fast and slow singularly perturbed subsystems. The resulting subsystem enables us to better understand the complex system and simplify the calculations. Powerful analytical, numerical, and asymptotic methods (e.g., the method of slow invariant manifolds and the homotopy analysis method) can subsequently be applied to each subsystem. In this paper, we compare the results obtained by the methods of slow invariant manifolds and SPVF, as applied to the spray (polydisperse) droplets combustion model.

1. Introduction

Mathematical models relating to various engineering applications are usually described by a large set of complex equations (differential equations). For the purpose of numerical, analytical, and qualitative analysis, it is often desirable to reduce the system to smaller subsystems with a comparatively small accuracy loss. Generally, a large set of differential equations describing a complex realistic phenomenon has a number of essentially different time scales (i.e., rates of change), which correspond to sub-processes. Given such systems, it is highly challenging to reveal the hidden hierarchy and implicit multi-time scales of the original systems that govern the equations; hence, an asymptotic method cannot be applied. Discovering the hierarchical structures of systems requires considerable complicated numerical treatments; however, this known hierarchical structure enables the application of numerous asymptotic approaches for the analysis of their behaviors.

Several asymptotic methods and numerical tools are available that can be applied to multi-scale systems. Examples include the method of slow invariant manifolds (MIM), which has been applied to the thermal ignition of diesel spray [1–4], the iteration method of Roussel and Fraser [5–8], the computational singular perturbation (CSP) method [9,10], geometric singular perturbation theory [11–13], and the intrinsic low dimensional method (ILDM), which is a numerical approach [14–17]. The ILDM method successfully locates slow manifolds in the considered system, but also exhibits several principal problems. The main constraints are as follows: the algorithm cannot be applied to phase space domains, where the RHS leading eigenvalues of the considered system Jacobian matrix are complex.

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In this case, the ILDM method does not produce any decomposition of the original system, or produces incorrect decomposition. Even in the case of an explicitly known decomposition, the ILDM cannot treat certain zones in the phase plane, such as the turning zones (manifolds); that is, zones where critical changes in the system behavior occur, and where the method numerical algorithm produces no additional relevant solutions to the system dynamics (ghost-manifolds). The transposition intrinsic low dimensional method (TILDM) is a modification of the ILDM method [18,19]. The TILDM is based on the geometrical approach to hierarchical systems of ordinary differential equations.

Researchers in the combustion theory field have suggested a new method known as global quasi-linearization (GQL) [20–23] and the singularly perturbed vector field (SPVF) to solve the above problems [24–27]. The main concept of this method is to transfer the original system of governing equations with the hidden hierarchy (hidden multi-scale structure) to its explicit form as the singularly perturbed subsystem (SPS). When this transformation from a hidden hierarchy model into a model with standard SPSs occurs, the analysis of the original system can be treated by the very powerful machinery of the standard SPS theory for model reduction and decomposition, as mentioned previously. The global information regarding the model decomposition is very useful and enables us to provide the qualitative structure of the slow and fast manifolds. This paper deals with the SPVF method applied to diesel spray thermal runaway.

The aim of this research is to expose the hierarchy of the combustion process in a diesel engine, and decompose the model into fast and slow subsystems. This decomposition enables us to apply the well-known asymptotic method of the invariant manifold. The reduction of the model into a subsystem simplifies the model and hence reduces the computation time.

2. Preliminaries to SPVF method

Given a large and complex mathematical/physical/chemical model with nonlinear governing equations, the aim of researchers in this field is to reduce the number of equations and discover the hierarchy of the dynamic system variables; that is, to decompose the system into fast and slow motions of the dynamic system variables. In order to achieve this, one should search for new coordinates and represent the governing equations (original model) in the form of a SPS. Once these coordinates are identified, we are able to decompose the original system into slow and fast subsystems, which enables us to apply asymptotic analytical methods such as MIM, perturbation analysis, and the homotopy analysis method (HAM). We introduce the general framework theory of the SPVF, as presented in the papers [28–30].

Definition 2.1. A real vector bundle ζ over a connected manifold $I\subset R^m$ consists of a set $E\subset R^m$ (the total set), a smooth map ρ: $E\to I$ (the projection), which is onto, and each fiber $F_x^{\zeta}=\rho^{-1}(x)$ is a finite dimensional affine subspace. These objects are required to satisfy the following condition: for each $x\in I$, there exists a neighborhood U of x in I, an integer k, and a diffeomorphism $\phi:\rho^{-1}(U)\to U\times R^k$, such that a homomorphism exists on each fiber ϕ.

Definition 2.2. Refer to a domain $V\subset R^n$ as a structured domain (or a domain structured by a vector bundle) if there exists a vector bundle ζ and diffeomorphism ψ: $V\to W$ onto an open subset $W\subset E$, where E is the total set of ζ.

Let

$$\frac{d\vec{x}}{dt}=\vec{F}(\vec{x},\delta), \tag{2.1}$$

for $\delta>0$ be a dynamical family of *ODE* systems.

Definition 2.3. Suppose that V is a domain structured by a vector bundle ζ and diffeomorphism ψ. For any point $\vec{x}\in V$, refer to $M_{\vec{x}}:=\psi^{-1}(\rho^{-1}(\rho(\psi(\vec{x})))\cap W)$ a fast manifold associated with point $\vec{x}$. Refer to the set of all fast manifolds M_x a family of fast manifolds of V.

Denote $dimM_{\vec{x}}=n_{fast}$ and refer to it as the fast dimension of $\vec{F}(\vec{x},\delta)$, and $TM_{\vec{x}}$ as the tangent space to $M_{\vec{x}}$ at point x.

Definition 2.4. A family of fast manifolds $M_{\vec{x}}$ is linear if there exists a fixed linear subspace L_{fast} of R^n, such that $M_{\vec{x}}=\{\vec{x}\}+L_{fast}$ for any $\vec{x}\in V$. Refer to L_{fast} as a fast subspace.

Definition 2.5. A parametric family $\vec{F}(\vec{x},\delta):V\to R^n$ of vector fields defined in a domain V, structured by a vector bundle ζ and diffeomorphism ψ, is an asymptotic SPVF if $\lim_{\delta\to 0}\vec{F}(\vec{x},\delta)\subset TM_{\vec{x}}$ for any $\vec{x}\in V$, and the structure of domain V is minimal for the vector field $\vec{F}(\vec{x},\delta)$ in the following sense. There is a proper vector sub-bundle ζ_1 of the vector bundle ζ, such that $\vec{F}(\vec{x},\delta)$ is not an asymptotic SPVF in domain V structured by the sub-bundle ζ_1 and the same diffeomorphism ψ.

This minimality property means that it is not possible to reduce further the dimension of fast manifolds $M_{\vec{x}}$ using sub-bundles. The next step is to introduce the method for decomposing the SPVF. Following the notations above, the vector field $\vec{F}(\vec{x},\delta)$ can be decomposed as the following sum:

$$\vec{F}(x,\delta)=\vec{F}_{fast}(\vec{x},\delta)+\vec{F}_{slow}(\vec{x},\delta), \tag{2.2}$$

where

$$\vec{F}_{fast}(\vec{x},\delta)=Pr_{fast}\vec{F}(\vec{x},\delta), \tag{2.3}$$