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3D mathematical model and numerical simulation for laying marine cable along prescribed trajectory on seabed

Huan Han[a], Xing Li[b], Huan-Song Zhou[a,*]

[a] Center for Mathematical Sciences and Department of Mathematics, Wuhan University of Technology, Wuhan 430070, China
[b] School of Mathematics and Physics, China University of Geosciences, Wuhan 430076, China

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A B S T R A C T

In this paper, we establish a mathematical model, that is, an initial-boundary value problem of partial differential equations (PDE), to describe the three dimensional (3D) motion of a marine cable being laid onto the seabed of varying depth. Based on this PDE model, a numerical method for simulating dynamically the moving process of marine cable is developed, which can be used to determine the cable laying operations in practice such that the marine cable can be laid on the prescribed position on seabed.

1. Introduction

As there is less and less exploitable petroleum resource onshore, more and more countries have begun to develop the offshore oil and gas since 1950s, and several large offshore oil fields were successfully discovered, one of them called Lula field was just discovered in 2007. It is prospected that the most possibility to find new big oil and gas fields is in ocean. So, how to explore effectively the undersea oil and gas resources becomes extremely important. During the last decades, marine seismic surveys are proven to be a safe and efficient technology in exploring for offshore oil and gas resources. The main idea of this kind of method is to make “earthquake” artificially in water (usually by air guns) to release sound waves which reflect off the subsurfaces of layers under the ocean floor, by analysis of the reflected waves (i.e. seismic data) from the seabed, geologist can use computer to image the geological structure of the target exploitation region and evaluate whether there contains energy resources beneath the sea floor. In order to collect accurately marine seismic data, we have to lay cables containing the sensors along the prescribed multiple parallel lines on the ocean bottom, see e.g. Fig. 1(a). However, this is not so easy because of the complicated ocean environment such as the currents, etc. see Fig. 1(b). The

* Corresponding author.
E-mail addresses: hanhuan11@whut.edu.cn (H. Han), lixing@cug.edu.cn (X. Li), hszhou@whut.edu.cn (H.-S. Zhou).

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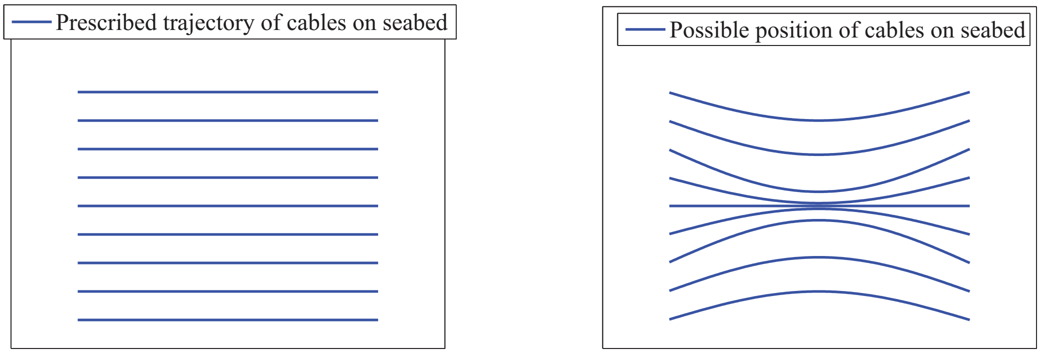

(a): Prescribed trajectory of cables on seabed **(b):** Possible position of cables on seabed without modifying the ship motion by considering the ocean effects

Fig. 1. Prescribed trajectory and possible position of cables on seabed.

present paper is to provide a mathematical analysis method which can be used in practice to design the sailing direction and trajectory of the cable laying ship such that the ocean bottom receiver cables can be broadly laid on the expected lines.

Nowadays marine cables being widely used in modern industry such as telecommunication, marine petroleum exploitation and power engineering, much attention has been paid to the theoretical studies about the dynamical and kinematical properties of marine cables during installation process. A systematic discussion about the force and motions of a marine cable in typical laying and recovery situation can be tracked back to the work of Zajac [1], since then many discussions on the behavior of marine cables for the laying practice, such as mathematical modeling, mechanical and numerical analysis, have been presented under variant ocean environment. Roughly speaking, there are two kinds mathematical models used in these discussions: consecutive models and discrete models. For consecutive models, e.g., [2–9], a cable is looked as a continuous and elastic mediums, which leads to PDE or variational models which are solved by difference methods. For discrete models, e.g., [1,5,10,11], a cable is divided into several rigid elements and their governing equations are derived from the principles of forces or moment equilibrium of the elements.

In the pioneering work of [1], Zajac made a good survey about the history of the laying of marine cable as well as the earlier literature on its dynamics and kinematics investigation, upon which he established formulas for the cable tension and cable position for 2D and 3D marine cable laying in stationary state by implicit integrals. Patel and Vaz [5] extended the 2D stationary state model of [1] to the dynamical state case and established a numerical model via Runge–Kutta method to study the transient behavior of marine cable approximately by dividing the cable into n line elements. Vaz and Patel [7] improved the 3D stationary model of [1] by additionally considering the elastic deformation of marine cable. Based on this result and the virtual work principle, Chucheepsakul and Srinil provided a variational model in [12,13] which makes it possible for applying the finite element method to analyze the 3D steady-state behavior of an extensible marine cable, and the Euler–Lagrange equations associated to this variational model are the same as the PDEs obtained in [7]. Recently, by using the flexible segment model (FSM), Xu et al. in [8,9] proposed a 3D dynamic calculation method for the cable tensions, which were generalized by Drag [10,11], etc. Moreover, there are also many papers concerning 3D models and numerical simulations for different types of marine cables based on the classical Newton's law and moment equilibrium principle, see, e.g., [2,3,14–20]. However, most of the papers mentioned above are mainly focussing on the cable tensions and cable angles, only a few of them may be use in some special cases to get the cable position information such as [5] for 2D case and [7,12,13] for 3D stationary case. In this paper, we are concerned with the 3D mathematical model which can be used in more general cases to predict dynamically the position of marine cable during the installation. For this purpose, the local coordinate system $\vec{t}, \vec{n}, \vec{b}$ used in [1,5–8] is not convenient. Here we use directly the global *Oxyz* system, see Fig. 3(a), where O denotes the origin, *Oxz* denotes the sea surface, *Ox* and *Oz* towards the east and the north, respectively. *Oy* towards the the seabed. This *Oxyz* system has two advantages:

- *Oxyz* system is aligned with the Global Positioning System (GPS), then the coordinate data, such as the depth of water $h(x, z)$, the ocean current velocity $[v_x(x, y, z, t), v_y(x, y, z, t), v_z(x, y, z, t)]$ and so on, can be directly obtained from GPS.
- The position information of the marine cable (inside water part) can be directly given by its coordinates (x, y, z) in the *Oxyz* system.

In this paper, we establish a 3D mathematical model of marine cable in the global coordinate system *Oxyz* by considering more ocean environmental effects such as lift force, drag force, ocean current and topography of seabed, etc. Particularly, our model can explicitly give the cable position information in the static case.