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Steady solution of solitary wave and linear shear current interaction

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ABSTRACT

The steady solution of a solitary wave propagating in the presence of a linear shear background current is investigated by the Green–Naghdi (GN) equations. The steady solution is obtained by use of the Newton–Raphson method. Three aspects are investigated; they are the wave speed, wave profile and velocity field. The converged GN results are compared with results from the literature. It is found that for the opposing-current case of the solitary wave with a small amplitude, the results of the GN equations match results from the literature well, while for the solitary wave with a large amplitude, results from the literature are seen to be not as accurate. In the following-current case, though the amplitude of the solitary wave is small, the GN results are shown to be accurate. The velocity along the water column at the wave crest and the velocity field for different cases are calculated by the GN equations. The results of the GN equations show obvious differences when compared with the results obtained by superposing the no-current results and linear shear current linearly. We find that for the same current strength, the vortex is stronger for the steep solitary-wave case than that for the small solitary-wave case.

1. Introduction

Many studies have focused on solitary waves based on the KdV equation. Some experiments were carried out by Longuet–Higgins [1] to test the accuracy of the KdV equation on steep solitary waves. Longuet–Higgins [1] found that the horizontal displacements of particles on the surface obtained by the KdV equation were much smaller than the experimental data. Umeyama [2] also found that the KdV equation was not very accurate in the study of the horizontal displacement of fluid particles after comparing the predictions with his laboratory measurements. Therefore, it is of interest to develop a better wave model to study steep solitary waves.

Solving Euler's equations to obtain the solitary-wave solution is regarded by some authors as an effective method. The algorithm of Tanaka [3] is widely used to solve Euler's equations. Dutykh and Clamond [4] proposed a more accurate method to obtain the steady solitary-wave solution of full Euler's equations. Their results showed that the calculations on steep solitary-wave profiles and velocity fields were accurate. For some more work done on solitary waves, see the introduction section of Dutykh and Clamond [4].

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The phenomenon of wave-current interaction always exists in coastal regions, therefore, theories that are based on the assumption of irrotational flow are limited in general although in some cases potential theory may be used when the vorticity of the background current is constant (see e.g., Touboul et al. [5]).

Some previous works have focused on waves in the presence of a uniform shear background current. Experiments were conducted by Soares and de Pablo [6] to study the influence of uniform current on the wave spectrum. Hsu et al. [7] studied the interaction between periodic waves and uniform current by the assumption of the potential flow, including the wave profile and particle trajectories. A new third-order trajectory solution in Lagrangian form for the water particles was developed by Hsu et al. [7]. Zhang et al. [8] and Zhang et al. [9] studied the propagation of periodic waves and solitary waves in the presence of a uniform shear background current by the Reynolds-averaged Navier–Stokes (RANS) equations, respectively. More recently, Duan et al. [10] studied the wave-current interaction in shallow waters by using the Green–Naghdi (GN) equations in the case of uniform current.

The interaction between a linear shear current and waves was also considered by some authors. The stream-function model was always used to study the interaction between the waves and linear shear current, such as by Dalrymple [11] and Vanden-Broeck [12]. In a previous work, Dalrymple [11] used a perturbation method to study the waves propagating in the presence of a linear shear background current and observed that the linear shear current would change the wave profile and wavelength for waves of the same height. Simmen and Saffman [13] focused on the interaction between the deep-water wave and linear shear current and considered both small-amplitude waves and nonlinear waves. The velocity was described by superposing the gradient of velocity potential and current velocity by Simmen and Saffman [13]. Swan [14] conducted some experiments to investigate the interaction between periodic waves and linear shear current. Different types of the linear shear current were used, while an ideal one could not be achieved easily in experiments by Swan [14]. Choi [15] investigated the nonlinear periodic-wave propagating in the presence of a linear shear background current, including the wave profile and the relationship between the wave speed and wave amplitude. A conformal mapping technique was used to derive the system of exact evolution equations for the free surface elevation and the free surface velocity potential by Choi [15]. Within the Lagrangian reference frame, Hsu [16] studied particle trajectories of the waves in the presence of a linear shear background current by a third-order solution.

Choi [17] studied a solitary wave in the presence of a linear shear background current by using an asymptotic model. Results given by Choi [17] showed that, in the opposing-current case, the wave profile was wider than that with no current; for the following-current case, the solitary wave profile was narrower than that with no current. Pak and Chow [18] derived a third-order solution by the perturbation expansion method to study a solitary wave in the presence of a linear shear background current. Their predictions included the results on the wave profile and wave speed. Since no higher-order solution than the third-order one was used to show convergence, for some strongly nonlinear cases, the accuracy of the third-order solution by Pak and Chow [18] has not yet been shown. Vanden-Broeck [12] used a boundary integral equation method to calculate the solitary wave in the presence of a linear shear current in waters of finite depth. Choi [19] used a set of nonlinear evolution equations derived under the long-wave approximation to study the effect of a background shear current on large amplitude internal solitary waves. Stastna and Lamb [20] studied the effect of background current on the properties of large solitary internal waves in a shallow, stratified ocean by using the fully nonlinear theory.

The wave-current interaction in the presence of an arbitrary shear current (both linear and nonlinear shear currents) was studied also. The previous work by Benjamin [21] was aimed at solitary wave propagation in the presence of a shear background current with an arbitrary vorticity distribution by using the stream function. Kirby and Chen [22] and Nwogu [23] showed that the current would change the dispersion relation of waves under the assumption of the arbitrary (vertical structure) current. Pak and Chow [18] also studied the nonlinear shear current cases, including the quadratic polynomial form and the index form. Some experiments were conducted by Swan et al. [24] to study the interaction of waves and depth-varying currents.

In this work, we use a strongly nonlinear theory, called the Green–Naghdi theory, originally developed by Green and Naghdi [25]. The governing equations in the GN equations are the depth-integrated form of Euler's equations. They satisfy the free-surface and bottom boundary conditions exactly. Distribution of the velocity field over the water column can also be viewed as prescribed by polynomial functions in finite-water depth according to Webster et al. [26]. Based on this, the GN equations are classified into different levels of equations. In the GN-1 equations, for example, the prescribed horizontal velocity distribution in the vertical direction is constant, while it is a linear and quadratic polynomial in the GN-2 and GN-3 equations, respectively.

Shields and Webster [27] discussed the solitary-wave solution of the GN-1, GN-2 and GN-3 equations. They found that the GN equations converged more rapidly than the ones based on perturbation methods, such as the one by Fenton [28]. More recently, Zhao et al. [29] obtained the steady solitary-wave solution of the high-level GN equations. They calculated the wave speed, wave profile, velocity field and particle trajectory. After comparing the results of the GN equations with results from the literature, it was shown by Zhao et al. [29] that high-level GN equations could provide a very good prediction for the steady solitary-wave solution.

Kim et al. [30] derived the Irrotational Green–Naghdi (IGN) equations from Hamilton's principle. The solitary-wave solution of the high-order IGN equations were given by Kim et al. [31], who studied celerity, mass and energy characteristics of a solitary wave. Recently, Zhao et al. [32] studied the particle-trajectory calculations under a solitary wave by high-level IGN equations. The results of the IGN equations were shown to match the experimental data very well on wave profiles and