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PHYSICAL MESOMECHANICS

Self-organized criticality of deformation and prospects for fracture prediction

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The paper considers common nonlinear characteristics of inelastic deformation and fracture of loaded solids and similarity of numerical solutions of a nonlinear system of relevant partial differential equations. The self-similarity of inelastic strain and damage accumulation in the entire hierarchy of scales — from interatomic distances up to tectonic faults of many thousands of kilometers in the Earth crust — ensures qualitative similarity of fracture scenarios whatever the scale of deformation and rheology of a medium. The common properties of deformed systems are spatial localization of inelastic strain and damage accumulation in the entire hierarchy of scales, further temporal strain localization as a superfast autocatalytic blow-up process, slow dynamics (deformation fronts or slow motions), and strain activity migration due to long-range space-time correlations over the entire hierarchy of scales. Thus, fracture evolves as a sequence of catastrophes of increasing scales up to macroscales. It is shown that self-organized criticality of any deformed system does not exclude the possibility to predict the time and the place of a future catastrophic event. Precursors of similar large-scale events can be (i) frozen strain activity in the immediate vicinity of a formed main crack or fault and (ii) generation of trains of deformation fronts (damage fronts) in this region and their flow toward the site of a formed main crack (fault).

Keywords: fracture prediction, nonlinear dynamics, damage accumulation, self-organized criticality, numerical simulation, blow-up modes, slow motion

1. Introduction. Physical mesomechanics and nonlinear dynamics

Almost 30 years has passed since Prof. Victor E. Panin and colleagues formulated the concept of structural levels of deformation [1, 2] which underlies the new scientific trend — physical mesomechanics of materials. In the framework of physical mesomechanics, its founder — V.E. Panin — and his followers and disciples have developed novel experimental and theoretical research methods and approaches to deformation and fracture of loaded materials and media as multiscale hierarchical systems whose evolution in effective force fields follows the laws of nonlinear dynamics or synergetics [3–18]. Synergetics as a self-organization theory and nonlinear dynamics as a whole have given an insight into many things, and primarily, into the general pattern of any evolutionary process. Particularly impressive works are those in which experimental data are processed by methods of nonlinear dynamics (certain of important examples are discussed below). However, the problem of research on the evolution of real natural and physical systems remains almost unsolvable, because their behavior is modeled by partial differential equations for which methods of analyzing common properties of solutions are hardly available. The only exception is a rapidly developed field of nonlinear dynamics — asymptology or asymptotic mathematics covering methods of asymptotic analysis of complex mathematical models of real objects [19].

Like nonlinear dynamics, physical mesomechanics actually treats the evolution of a specific nonlinear system — a loaded solid [15, 16, 20, 21] or a hard solid medium, e.g., a geological medium [21, 22]. In this sense, theoretical physical mesomechanics is thus part of nonlinear dynamics. All laws of evolution of complex nonlinear systems in classical nonlinear dynamics or synergetics are found in full measure in loaded solids and solid media. This is evident from both experiments [13, 16, 23] and theoretical solutions of evolution equations for a deformed solid in effective force fields [15, 20]. Nonlinear dynamics investigated and continues for the most part to investigate the peculiari-

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doi:10.1016/j.physme.2010.11.010

ties of solutions of base synergetic equations, such as heat or diffusion equations, Ginzburg–Landau and Korteweg–de Vries equations, cubic Schrödinger equations (and its other forms), and simpler differential equations that admit of analytical analysis of peculiarities of their solution [24–31]. However, attempts to employ these base equations in analysis of the behavior of real physical and natural systems or in applied research are very often vain and incorrect for the mere fact that these equations are mostly not a strict mathematical model of the phenomenon under study. The similarity of the basic features and peculiarities of evolution scenarios — a threshold character of physical phenomena, spatial-temporal localization of processes, fast modes of evolution, bifurcations and changes of evolution scenarios, correlative behavior of system elements in a wide range of scales, self-organization, and many others — is by no means an indication of the similarity as such between evolution scenarios of various nonlinear systems. The scenarios are most likely to differ radically; a classical example is the sensitivity of many nonlinear systems to the least variation of equation coefficients resulting in distinct evolution scenarios in the systems. So a slight variation in coefficients in a trimolecular kinetic model — a Brusselator — can give us rings, spirals, multiturn spirals, etc. [29, 30], i.e., even in the same equation, the variation in specified properties of a nonlinear medium results in different types of dissipative structures, their multidimensional architecture and evolution in time [31].

For correct theoretical study of the evolution of any real process, we should first of all construct its realistic mathematical model, formulate an evolutionary problem, and analyze solutions of appropriate nonlinear equations [20, 22] rather than resort to well-studied base equations for the mere reason that their solutions demonstrate, for example, the observed blow-up effect or localization of parameters. Clearly with rare exception, this will be systems of partial differential equations admitting only of their numerical solution. The need for analysis of numerical solutions of partial differential equations as equations describing the evolution of real physical processes or giving a solution of applied problems was ripe long ago and is one of the most urgent problems of nonlinear dynamics. In our works [20, 22, 32], it is shown that solutions of partial differential equations of solid mechanics demonstrate all peculiar features of the evolution of dynamic nonlinear systems (they are listed above) where the dynamic problem to be solved is stated as an evolutionary problem [20]. The use of well-studied base synergetic equations by many researchers in analysis of real natural and physical systems allows little other than familiar general reasoning about peculiarities of an evolutionary process, fails to give quantitative characteristics of the evolution, and generates sceptisism in the scientific community about the potentialities and efficiency of nonlinear dynamics methods.

The new paradigm of physical mesomechanics has been found so fruitful that despite the tremendous successes and new understanding of deformation and fracture mechanisms, it opens up more and more avenues for study of urgent strength problems of materials and constructions in the framework of its concepts and approaches.

One of these "perennial problems" is the prediction of the place and time of future fracture, be it fracture of a laboratory specimen or a structural member, a catastrophic mine roof collapse, or a fault in the Earth crust that leads to an earthquake. It is thought that the paradigm of physical mesomechanics — consideration of a deformed medium as a multiscale nonlinear system — is bound to radically change both the understanding and the approaches to the solution of this problem.

This paper is a theoretical analysis of the evolution peculiarities of deformed nonlinear systems in the framework of the mathematical evolutionary theory of loaded solids and media developed by the author and colleagues [20, 32]. The numerical solutions to be analyzed are those of dynamic equations of solid mechanics.

2. Fracture prediction and self-organized criticality of deformed nonlinear systems

If we put the simple, but very important question of whether there exits an easy and efficient procedure to predict the place and time of fracture or grounds to hope for its development in the nearest future, the answer to the first part of the question will be "no" and to the second part — "yes".

The traditional criterion approach of phenomenological fracture macromechanics is incapable, in principle, of solving the prediction problem. According to the traditional notion, for example, of the σ–ε diagram of a deformed metal polycrystal (Fig. 1), we first observe various stages of strain hardening, and as the ultimate strength is reached, fracture occurs. A similar interpretation is conventional also for fracture of brittle materials with the only difference that the small portion of the ascending branch of the stress–strain curve after elastic deformation is interpreted as inelastic deformation caused by the medium compliance in response to damage accumulation. The question arises of how a damaged medium can be stronger than a less damaged one at lower inelastic strain. The same question can be addressed to strain hardening. The growth of the mean stress over a specimen for both plastic and brittle materials has no relation to its actual local strength under loads; this strength is much higher than the specimen mean macrostress described by the σ–ε diagram. The local strength of a loaded medium can only drop due to accumulation of micro- and mesoscale defects and damages of varying physical origin in the medium. When a slow quasistationary stage of damage accumulation gives way to a superfast autocatalytic process — a blow-up mode in a local fracture region — the local