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Delayed fracture of plates under creep condition in unsteady complex stress state in the presence of aggressive medium

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ABSTRACT

Delayed fracture of bended rectangular plate is researched under creep condition in unsteady complex stress state considering the influence of the ambient medium. Using Rabotnov's kinetic theory, time to fracture of such plate is determined during sequential bending in different planes. Piecewise constant dependence of the bending moment direction and level on time is considered. The time to fracture of the plate is determined using a linear fractional creep model. The effect of the ambient medium on the creep and the creep rupture of the plate is attributed to diffusive penetration of the ambient medium elements into the material of the plate. An approximate method based on the introduction of a diffusion front is used to evaluate the diffusion process rate. The ambient medium effect is taken into consideration by introducing the function of cumulative average concentration into the constitutive and kinetic linear fractional equations. The times to fracture for the scalar and vector damage parameters are compared.

1. Introduction

Real structures can be used under broadly varied conditions and in various media, which are quite often somewhat aggressive to the material of the structure. Such media may be natural, anthropogenic, or mixed. Certain aggressive components of these media may diffuse into the material and enter into physical and/or chemical interaction with it. This interaction will lead to changes of the service properties of the material (dielectric, thermophysical and physical-and-mechanical ones) and of the entire structure.

Forecasting durability of materials and structural elements under long-term high-temperature stress in the presence of an aggressive ambient medium is an extremely important problem to ensure reliability during the entire operational life. Structural elements made of metal alloys and resting under high temperatures for a long time are usually exposed to creep and delayed fracture processes [1]. A rectangular plate is one of the most common structural elements, and determining its time to fracture under the above-described conditions is both fundamental and applied by nature when calculating structural elements in power and aviation-and-space industries. The issues of creep and creep rupture strength with due consideration of the ambient medium lack researchers' attention. Among the scientists engaged with such research, there should be noted the Russian authors Ovchinnikov and co-workers [2–5], Kiselevsky [6], Pavlov et al. [7], Lokoshchenko and co-workers [8–12] and also some foreign authors, e.g. [13–15]. The papers, as a rule, consider uniaxial extension. By this

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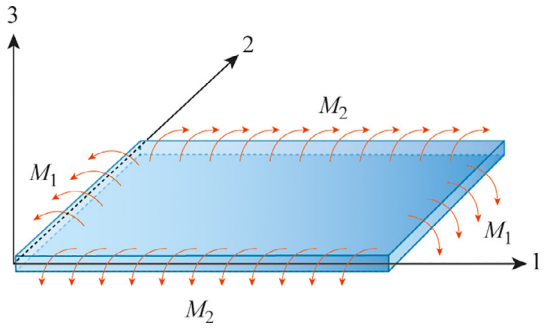

Fig. 1. Loaded plate.

reason, the delayed fracture research considering the effect of the aggressive medium under the unsteady complex stress condition in this work is undoubtedly vital at present.

In this paper, the effect of the aggressive medium is attributed to diffusive penetration of medium elements into the material of the plate reducing its service durability.

Differential equations of parabolic type are used to solve a lot of diffusion [14], filtration [15], and heat conductivity [16] problems. Naturally, they are subject of extensive research. It is of interest to consider the methods that allow obtaining qualitative and quantitative description of changes of a given characteristic in time and space in a visible form.

Exact solutions of the diffusion equation for restricted bodies are usually represented by trigonometric series or series comprising special functions, which do not always allow presenting the required parameters in a transparent, analyzable form. These solutions are tedious by form and, besides, to obtain a satisfactory computing precision, especially at small times, it is necessary to keep a large number of series members. When solving equations with time-varying boundaries of the domain under consideration, especially if such a dependence should be determined from the solution, it is reasonable to use approximate methods.

Approximate methods of time-varying fields calculation are described in a number of classical monographs [17–19] and others. The majority of the reviewed methods present the parabolic equation solution in a form of a polynomial with respect to spatial coordinates. Whereas the coefficients of the series members are either constant or time-varying and are determined with the help of various variation methods.

Some scientists believe it is reasonable to divide the entire domain under consideration in disturbed and non-disturbed parts and study the motion of the boundary (diffusion front) between these parts. It appears that the method of approximate solutions of parabolic problems associated with the introduction of a disturbance front was first considered by Lembke [20], then further developed by Leibenzon [17], Charny [21], Barenblatt [22], Shesterikov and Yumasheva [23] and other scientists.

During the recent years, Lokoshchenko and his students have been carrying out a systematic experiential-and-theoretical research of the aggressive medium effect on the creep and creep rupture of metals [8–12, 24–31]. The research resulted in obtaining solutions to problems when the diffusion front was interacting with the disturbance front. It was shown that the diffusion front coordinate depended not only on the distance to the external boundary, but also on the curvature of the boundary; inaccuracies of the obtained solutions were evaluated. Probability-based theory of creep and creep rupture strength was suggested. An approximate method was used to solve the problems of extending, bending, complex unsteady stress state in the rods and plates under creep conditions in an aggressive medium.

2. Problem statement and research purpose

Under research, there is delayed fracture of a rectangular plate of thickness H that is exposed to bending moments M_1 and M_2 distributed along its edges. Their dimension is $[M] = Nm/m = N$(Newton). A diagram of the loaded plate is given in Fig. 1.

The plate is placed in an aggressive ambient medium. A linear diffusion process along axis 3 (thickness) symmetric with respect to the middle plane of the plate is considered. Based on kinetic creep and creep rupture theory by Rabotnov [1], the problem of determining the time to fracture of the plate under different combinations of the bending moments distributed along the edges is stated and solved.

The novelty of the suggested scientific research lies in the use of the linear fractional creep and creep rupture strength model, whose advantages compared to the widely-spread power law model are shown in the next section of the article.

3. Linear fractional creep model

The researchers widely use the concept of mechanical constitutive equations in the creep theory. Therefore, much attention is paid not only to building general equations for non-elastic media, but mainly to selecting specific rheonomic dependences describing the behavior of the materials under stress. A peculiarity of building such type of dependences is that the constitutive equations should not only describe the experimentally obtained phenomena but should be acceptable