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# Mutual effect of Coriolis acceleration and temperature gradient on the stress and strain field of a glass/epoxy composite-pipe

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ABSTRACT

Rotating composite pipes due to their light-weight, high specific strength and stiffness and excellent fatigue resistance are a good substitute for most common metallic alloys. This paper presents the exact three-dimensional thermoelastic solution for a composite pipe using the heat transfer equation along thickness, by taking into account the body forces created by rotation and the Coriolis effect. A Bessel nonhomogeneous differential equation is derived for displacement along the radial direction, which can be approximated by a Euler equation using the pipe wall thinness restriction. The composite pipe rotational speed and wall thickness are two key parameters which are used to illustrate the Coriolis effect on the pipe stress and strain filed. Results indicated that for the wall thickness to inner radius ratios less than 0.1, all stresses, strains, and displacements were experienced 4% increase, except for radial stress, upon which Coriolis acceleration *had no effect*. In the case of thermomechanical loading, increasing the pipe wall thickness resulted in variation of the thermal stress and the centrifugal force values. Numerical verifications showed that for thickness to radius ratios less than 0.05 the amount of deviation between Bessel (numerical) and Euler (analytical) formulation is less than 5%.



## 1. Introduction

Higher specific stiffness and strength, resistance to corrosion and lower weight compared to metals have resulted in polymeric composites wide and constantly growing application in different industries. Advanced polymeric composites are widely used to produce rotating composite shafts for different application such as drive shaft. Composite pipes experience different types of mechanical loading in service including temperature gradient. Some studies regarding the mechanical properties and the failure of composite pipes under bending load [1,2], transverse load [3,4] and axial pressure [5,6] have been carried out. Because of their importance in engineering applications [7–9] such as power transmission shafts, rotating structures are a noteworthy field of research. Ben [10] reported the thermal stresses and deformations in a thick-walled cylindrical pipe using the finite element method. Xia et al. [11] developed an analytical solution to determine thermal stresses in a sandwich thick-walled pipe under internal pressure and thermomechanical loading, making use of the classical laminated-plate theory. An analytical solution based on the theory of elasticity for multilayered pipes under internal pressure has been presented by Xia et al. [12]. In this study, three different lay-ups with similar mechanical properties were

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examined to determine the effects of different lay-ups on stress and strain components in cylindrical coordinates. Bekaiyan et al. [13] continued Xia's work and applied a temperature gradient along pipe wall thickness, in addition to internal pressure, to the composite pipe. Ansari et al. [14] carried out stress analysis of filament-wound composite pipes under cyclic internal pressure and temperature loadings based on the theory of elasticity and used a numerical solution to obtain the time-dependent stress, strain, and displacement. A study on rotating disks subject to internal and external pressure as well as temperature distribution was carried out by Mohammadi et al. [15]. He used the modified Tsai-Wu failure criterion and proved that failure depends strongly on temperature distribution. Aksoy et al. [16] analyzed the two-dimensional thermoelastic stress in pipes laminated with isotropic materials. He used the plane strain condition and took advantage of the stress function to determine stresses. Chen and Lee [17] carried out the analysis of a rotating composite pipe with limited length simply supported at both ends. He considered the Coriolis effect and employed the state-space approach, and the layerwise method used the Fourier expansion technique to obtain a solution. Liu and Chen [18] carried out an analysis of a piezoelectric rotating pipe and considered the Coriolis effect in the analysis of thin-walled pipes.

The goal of this study is to obtain the stress and strain distribution in cylindrical coordinates for an anisotropic rotating composite pipe under the temperature gradient. The three-dimensional theory of elasticity has been used to find the solution while considering the body forces caused by rotation and the Coriolis effect. Knowing the coefficients of conductions and convection in the inner and outer surfaces of the pipe, solving the heat transfer equation would reveal the temperature distribution along the composite pipe thickness. The resulting differential equation for radial displacement is of the Bessel type, which, with the assumption of a thin-walled composite pipe can be approximated by a Euler differential equation and thus can be solved.

## 2. Analytical solution

### 2.1. Equation of conduction

The temperature gradient is one of the most pivotal factors which should be considered in the design of composite materials. Changes in temperature result in different variations in length along various directions of the composite layers which are perfectly bounded to each other, lead to the formation of thermal stresses in composite materials.

The general form of heat transfer equation in cylindrical coordinates is as follows [19]:

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial x^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{1}$$

Where $\dot{q}$ is the rate of internal energy generation, $k$ is the coefficient of thermal conduction and $\alpha$ is the coefficient of thermal diffusivity.

Due to the pipe being long, its symmetry and steady state heat transfer, temperature distribution will be independent of both $x$ and $\theta$. Also, as there is no heat generation, Eq. (1) reduces to:

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} = 0 \tag{2}$$

Integrating Eq. (2) yields:

$$T = A + B \ln r \tag{3}$$

The inner side of the pipe is in contact with the hot fluid $T_f$ and the outer side with the surrounding air $T_a$:

$$-k\frac{\partial T}{\partial r} = \bar{h}_i(T_f - T) \qquad r = r_i \tag{4a}$$

$$-k\frac{\partial T}{\partial r} = \bar{h}_o(T - T_a) \qquad r = r_o \tag{4b}$$

Where $r_i$ and $r_o$ are the pipe inner and outer radii, respectively $\bar{h}_i$ and $\bar{h}_o$ are the mean convection heat transfer coefficient for the inner and outer sides of the pipe, respectively. Integrations constants $A$ and $B$ are calculated as follows, according to boundary conditions:

$$A = \frac{k\left(\dfrac{T_f}{r_o\bar{h}_o} + \dfrac{T_a}{r_i\bar{h}_i}\right) + (T_f \ln r_o - T_a \ln r_i)}{k\left(\dfrac{1}{r_o\bar{h}_o} + \dfrac{1}{r_i\bar{h}_i}\right) + \ln(r_o/r_i)} \tag{5a}$$

$$B = \frac{T_a - T_f}{k\left(\dfrac{1}{r_o\bar{h}_o} + \dfrac{1}{r_i\bar{h}_i}\right) + \ln(r_o/r_i)} \tag{5b}$$