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Enhanced linear reformulation for engineering optimization models with discrete and bounded continuous variables

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ABSTRACT

In this paper, we significantly extend the applicability of state-of-the-art ELDP (equations for linearizing discrete product terms) method by providing a new linearization to handle more complicated non-linear terms involving both of discrete and bounded continuous variables. A general class of "representable programming problems" is formally proposed for a much wider range of engineering applications. Moreover, by exploiting the logarithmic feature embedded in the discrete structure, we present an enhanced linear reformulation model which requires half an order fewer equations than the original ELDP. Computational experiments on various engineering design problems support the superior computational efficiency of the proposed linearization reformulation in solving engineering optimization problems with discrete and bounded continuous variables.

1. Introduction

For real-world engineering applications, there has been increasing attention focused on model-based methodologies. The modeling of engineering optimization problems usually leads to a non-linear program in which some or all of the variables are restricted to a set of discrete values. Typical applications involving the use of non-linear discrete programming can be found in aerospace structural analysis [1], topological decision making [2], chemical process synthesis [3], systems reliability design [4], process design [5] and engineering design [6,7]. Effective non-linear discrete optimization methods then become crucial for obtaining viable solutions to practical engineering optimization problems.

It is well known that non-linear discrete programs are NP-hard in general. The difficulties in efficient computation arise in the combinatoric nature of discrete variables and the non-linearity and non-convexity embedded in the objective and constraint functions [8]. In the past decades, heuristics techniques [9] and branch-and-bound based algorithms [10] have been extensively studied for finding approximate and exact solutions to non-linear discrete programs, respectively. In the simple case of mixed-integer linear programming (MILP), branch-and-bound based commercial solvers such as CPLEX and Gurobi are available for finding exact solutions in an efficient manner.

This paper focuses on a specific, but wide, class of non-linear mixed-integer optimization problems called representable programming problems [11], which involve a set of constraints in both discrete and bounded continuous variables. Representable programming problems naturally occur in many real-life engineering applications such as the heat exchanger

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network synthesis [12], chemical equilibrium problem [13], design of a reactor network [14], gate sizing in circuits [15], and three-stage process system with recycle [16]. An important subclass of representable programming problems with only discrete variables is the polynomial discrete optimization problem with exemplary formulations from Alkylation process optimization [17], water pumping system [13] and design of a welded steel beam [2].

Note that polynomial discrete programming remains to be NP-hard. Its complexity was particular studied in [18]. To globally solve polynomial discrete programs via branch-and-bound schemes, relaxation methods have been explored for providing effective lower bounds [19–22]. In some special case such as cubic discrete programming, optimality conditions have been explicitly derived [22]. Another important class of solution methods transforms a polynomial discrete program into a 0–1 MILP problem and solves the reformulation via MILP solvers [23–28]. Taking computational advantage of commercial solvers, linear reformulation techniques have been widely adopted for solving large-scale polynomial discrete programs in practice.

Regarding computational efficiency of solving a polynomial discrete program via 0–1 MILP reformulation, much effort has been spent on reducing the required number of variables and linear constraints in the reformulation model. In addition, the computational effectiveness of conventional reformulation models may suffer from unbalanced branch-and-bound trees [25]. In resolving this issue, Li et al. [27] proposed a set of equations for linearizing discrete cross-product terms (ELDP) that can be incorporated into conventional MILP reformulations of polynomial discrete programs. The ELDP linear reformulation model has been shown to exhibit balanced branch-and-bound trees, and achieve a two-order reduction in computational time for finding an exact solution.

Motivated by the ELDP linear reformulation model of polynomial discrete programs, we intend to extend ELDP method to linearize more general optimization problems with more complicated cross-product terms with both of the discrete and bounded continuous variables. In particular, we explore the applicability of ELDP method to solving representable programming problems with wider engineering applications. To further enhance ELDP method for efficiently solving engineering optimization problems, we investigate the logarithmic feature embedded in the binary representation of discrete variables to derive a new linear reformulation using a minimum number of linear constraints. The enhanced ELDP linear reformulation requires half an order fewer equations for linearizing general discrete terms and restricts the values of discrete terms more effectively, which may lead to more balanced branch-and-bound trees. Results of computational experiments on literature-reported engineering design problems are provided to support the superior computational performance of the proposed enhanced ELDP linear reformulation model.

With insightful theoretical analysis and supportive numerical results, this study provides a powerful reformulation model with desired applicability and efficiency for solving engineering optimization problems involving discrete and bounded continuous variables. We significantly extend the ELDP method to handle a wider range of engineering optimization problems, including those represented by geometric discrete programs and fractional discrete programs. With enhanced ELDP linear reformulation model, large-scale engineering problems formulated as non-linear discrete programs can be efficiently solved.

The rest of this paper is organized as follows. Section 2 reviews the basics of ELDP method for solving polynomial discrete programs. Section 3 extends ELDP method to treat more complicated terms and thus to linearly reformulate the representable programming problems. Section 4 proposes an enhancement technique to further reduce the number of linear constraints required in the ELDP linear reformulation model. Section 5 conducts numerical experiments on engineering optimization problems to highlight the computational effectiveness of the enhanced ELDP method. Section 6 concludes this paper.

2. ELDP method for polynomial discrete programs

In this section, we briefly review ELDP method [27] that was proposed for fast computation of solving polynomial discrete programs. Consider the following simple cubic polynomial discrete program:

P (cubic polynomial discrete problem)

$$\text{Min} \quad F_0(\boldsymbol{x}) = \sum_{i=1}^{n} a(0)_i x_i + \sum_{i=1, j\geq i}^{n} b(0)_{i,j} x_i x_j + \sum_{i=1, j\geq i, k\geq j}^{n} c(0)_{i,j,k} x_i x_j x_k \tag{1}$$

$$\begin{aligned} \text{s.t.} \quad & F_v(\boldsymbol{x}) = \sum_{i=1}^{n} a(v)_i x_i + \sum_{i=1, j\geq i}^{n} b(v)_{i,j} x_i x_j + \sum_{i=1, j\geq i, k\geq j}^{n} c(v)_{i,j,k} x_i x_j x_k \geq C_v \\ & \text{for } v = 1, \ldots, V, \\ & x_i \in \{d_{i_1}, d_{i_2}, \ldots, d_{i_m}\} \text{ for } i = 1, \ldots, n, \end{aligned} \tag{2}$$

where the coefficients $a(v)_i, b(v)_{i,j}, c(v)_{i,j,k} \in \mathbb{R}$ for $i,\ j,\ k = 1, \ldots, n,\ i \leq j \leq k$ and $v = 0, \ldots, V,\ C_v \in \mathbb{R}$ for $v = 1, \ldots, V,$ and each variable x_i is assumed to have m possible discrete values $\{d_{i_1}, d_{i_2}, \ldots, d_{i_m}\}$ with $d_{i_1} < d_{i_2} < \ldots < d_{i_m}$ for $i = 1, \ldots, n$. To develop a 0–1 MILP reformulation of problem **P**, the basic framework is to linearly express the discrete variables x_i and discrete cross-product terms $x_i x_j$ and $x_i x_j x_k$, using only binary variables and continuous variables.

Remark 1. With some adaptations, ELDP method can handle polynomial discrete programs in higher orders. For simplicity, the treatment of a cubic polynomial discrete program is discussed in this section.

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