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Analytical solution for nonlocal coupled thermoelasticity analysis in a heat-affected MEMS/NEMS beam resonator based on Green–Naghdi theory

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ABSTRACT

In this article, an analytical solution is presented for coupled thermoelasticity analysis (with energy dissipation) in a micro/nano beam resonator, considering small scale effects on the transient behaviors of fields' variables. The Green–Naghdi (GN) theory of generalized coupled thermoelasticity and nonlocal Rayleigh beam theory (NRBT) are employed to derive the temperature and lateral deflection in the closed forms. The presented analytical solution is based on Laplace transform. To find the dynamic and transient behaviors of fields' variables in time domain, an inversion Laplace technique is utilized, which is called Talbot method. The effects of some parameters such as small scale parameter and dimensions of the beam on the dynamic behaviors of temperature and lateral deflections are discussed in details. The propagation of wave fronts in both temperature and lateral deflection domains are obtained and graphically illustrated at various time instants.

1. Introduction

The micro-electro-mechanical systems (MEMS) or nano-electro-mechanical systems (NEMS) have been attracted in many of applications in engineering based on its capabilities in small scales. One of the micro/nano structures, which are commonly used in many technological applications, is beam resonator. The micro/nano scale beam resonators response to excitations very fast with a high sensitivity. Consequently, it is very important to simulate the dynamic behaviors of MEMS/NEMS subjected to shock loadings including mechanical or thermal shock loadings. Some applications of MEMS beam resonators have been reported as scanning probe microscopy, vibration shock sensors, filters, resonant sensors, actuators, gyroscopes and other sensors [1–6]. The coupled thermoelasticity analysis in micro/nano beam resonators should be taken into account when the resonators are subjected to a mechanical or thermal shock loading. One of the most important problems in transient and dynamic behaviors of temperature and elastic fields is the energy dissipation (thermoelastic damping). There are some research works in which the thermoelasticity analysis of micro or nano beams are modeled using the theories of generalized coupled thermoelasticity.

To have a realistic analysis of MEMS/NEMS beam resonator, the transient behaviors of temperature field should be studied using a convenient thermoelasticity theory in which the interactions between various fields such as temperature and displacements fields are taken into account. These kinds of thermoelasticity theories are referred as the coupled thermoelasticity theories, which are divided to two main sets, which the first kinds are the classical coupled thermoelasticity theories

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and the second ones are the non-classical coupled thermoelasticity theories. In the classical theories of coupled thermoelasticity, the propagations of thermal and elastic waves are predicted by the infinite speeds, which is not a realistic assumption. In this regards, the non-classical theories of coupled thermoelasticity have been developed to simulate the wave propagations with finite speed. In the non-classical theories of coupled thermoelasticity, the interactions between elasticity field (displacement field) and temperature field are considered in the simulations and also a finite speed is admitted for the propagation of thermal waves through the continuum, which are called the second sound. The theory of generalized coupled thermoelasticity with one relaxation time for the special case of an isotropic body was presented by Lord and Shulman [7] and with two relaxation times was proposed by Green and Lindsay [8].

One of the most important coupled thermoelasticity theories was presented by Green and Naghdi as GN theory in which the propagation of thermoelastic waves can be modeled in a domain with high-rate excitation [9–11]. The GN theory of coupled thermoelasticity is based on the some effective basic modifications in the constitutive equations that covers a much wider of coupled thermoelasticity cases. For example, the type I of GN theory covers classical theory, the type II of GN theory covers the coupled thermoelasticity without energy dissipation and then the type III of GN theory simulates the coupled thermoelasticity with energy dissipation.

The thermoelastic damping in the microplates was studied by Nayfeh and Younis [12–14] using the analytical solutions. The thermoelastic damping of silicon-based MEMS beam resonators was studied by Houston et al. [15] in which the significance of energy dissipation was presented by them. The effects of energy dissipation and thermoelastic damping on the frequency shift ratio was obtained and studied by Guo and Rogerson [16] using a 2D parabolic heat conduction model. Employing the model order reduction, Choi et al. [17] presented a finite element formulation of fully coupled thermoelastic problem. Zamanian and khadem [18] modified the classical theory by considering stretching of neutral axis of bending. The energy dissipation in a capacitive micro-beam resonator was studied by Vahdat and Rezazadeh [19] using finite element method and two dimensional hyperbolic heat conduction model with one relaxation time. An analytical solution was presented by Sun et al. [20] to study on the thermo-mechanical behaviors of a micro beam. They assumed that the micro beam was excited by a pulsed laser heating. To find the thermoelastic damping in the micro beam, they employed the Lord-Shulman theory of coupled thermoelasticity without considering the small scale effects of micro beam on the results. A combined computational method based on the state-space and Laplace transform methods was presented by Elsibai and Youssef [21] for coupled thermoelasticity analysis of a gold nano beam resonator subjected to ramp type heating. They employed the Green–Naghdi theory of generalized coupled thermoelasticity without energy dissipation and without considering small scale effects for their research. The effect of the ramping time parameter on the dynamic behaviors of fields' variables was presented for gold nano-beam in femtoseconds scale. Another analytical solution based on an eigenvalue approach and Lapace transformation technique was presented by Abbas [22] for coupled thermoelasticity analysis in a micro beam. He derived the governing equations of the problem based on the Green–Naghdi theory without considering the small scale effects of micro beam. The uncoupled thermoelasticity analysis and Fourier heat conduction in the MEMS/NEMS were carried out by Zozulua and Saez [23] using the Fourier series expansion in terms of Legendre polynomials. They solved an uncoupled thermoelasticity in MEMS/NEMS without considering small scale effects. Abbas [24] presented an analytical solution for coupled thermoelasticity and the thermoelastic damping analysis in the NEMS using the two-temperature model of coupled thermoelasticity, which is based on the Lord-Shulman theory of the generalized coupled thermoelasticity. In his work, the size effects were not considered to derive the governing equations and analytical solution. Zuo et al. [25] proposed an analytical solution based on the series solution for thermoelastic damping analysis in the bilayered fully clamped rectangular and circular micro plates. An analytical solution was proposed for coupled diffusion-thermoelasticity analysis in a nano beam resonator as NEMS by Alzahrani and Abbas [26] using the Lord-Shulman theory of the generalized coupled thermoelasticity. They solved the problem without considering the small scale effect using the Laplace transformation technique. The thermoelastic damping in a micro ring resonator with irregular mass and stiffness has been studied by Kim and Kim [27] without considering the small scale effects. They used the coupled Fourier's one dimensional heat conduction to analytically derive a formulation for quality factor. In another work, the thermoelastic damping and coupled thermoelasticity were studied by Li et al. [28] in a microring using an analytical model for quality factor. They assumed that the micro ring has a circular cross section and the coupled heat conduction and elasticity equations (without considering small scale effect) were analytically solved in a two dimensional case. In 2017, Partap and Chugh [29] presented an analytical solution for coupled thermoelasticity analysis based on the Lord-Shulman theory in a micropolar microstretch beam resonator (without considering small scale effect). They used the Laplace transforms technique to solve the governing equations analytically. An analysis on the thermoelastic damping based on the classical theory of coupled thermoelasticity in a FG micro beam with rectangular cross section was performed by Li et al. [30] using an analytical method. The governing equations in both temperature and deflection fields in FG micro beam were derived by them without considering the small scale effects. An explicit formula was proposed for thermoelastic damping in nano beam resonator made of gold by Youssef and El-Bary [31] considering the reference temperature dependent Young's modulus. They derived their formulation using Lord-Shulman theory of the generalized coupled thermoelasticity without using theories for considering small scale effects. The quality factor of thermoelastic damping was analytically expressed by Liu et al. [32] for a bilayered circular plate resonators with two-dimensional heat conduction. They employed the classical theory of coupled thermoelasticity to derive the governing equations without considering small scale effect. A coupled thermoelasticity analysis based on Lord-Shulman theory was presented by Partap and Chugh [33] to study on thermoelastic damping in a rectangular micro plate without considering small scale effect using an analytical approach.