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Energy-efficient timetable and speed profile optimization with multi-phase speed limits: Theoretical analysis and application

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ABSTRACT

With ever-increasing energy consumption and associated costs, energy-efficient timetable and speed profile optimization approaches for metro systems have attracted much attention from both academic and industry communities. This paper develops an integrating timetable and speed profile optimization model with multi-phase speed limits to reduce the energy consumption for a metro line. First, the headway and cycle time for the next train are determined based on the passenger demand; and the optimal coasting-switching points on each section are theoretically analyzed with consideration of the multi-phase speed limits, headway and cycle time constraints. Second, the non-convex train scheduling problem is transformed to a strictly quadratic model by using the Taylor approximation. Finally, we apply the active set method (ASM) to find the approximate optimal solution and present numerical examples with real-world operational data from the Beijing Metro Yizhuang Line in China. The results shows that the developed approach can reduce the energy consumption by 4.52% for one train finished one round trip in comparison with the current timetable.

1. Introduction

Metro systems play an important role in urban economic and social development for metropolis due to the high loading capacity, punctuality and low pollution. In China, almost all big cities are building metro lines to relieve the traffic pressure. For example in Beijing, it has 19 lines with a total length of 574 km until now. The Yizhuang Line is a short metro line in Beijing only including 12 stations, which consumes about 42.53 million kW•h of electricity per year. This amount of consumed electricity is equivalent to the demand of 22,000 dwelling homes in Beijing. We can imagine that the total energy consumption of the whole metro system per year is a considerable magnitude. As a key operation element, a well-designed train timetable and speed profile can improve the energy performance of the metro system.

The majority of studies on train timetable mainly focused on the delay management [1], balance passengers service [2], timetable coordination [3] and train dispatching [4]. We can refer to Corman and Meng [5] for the surveys that focus on the train-scheduling problem in rail or metro systems. Recently, with increasing costs of energy resources and carbon emissions, energy conservation has become a hot topic and received much attention from almost all industries including

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metro or rail operations [6–9]. In general, due to section running times for trains having a direct influence on the traction energy consumption, the energy-efficient speed profile of trains operating a trip unless the track contains steep grades composes of three working phases: maximum acceleration, coasting and maximum deceleration [10]. But for a long-distance or a multi-phase-speed-limit journey, an additional speed-hold stage (i.e., cruising) should be considered for the energy-efficient speed profile to satisfy the constraint of running time. Thus, the optimal train control strategy incorporates four phases correspondingly: maximum acceleration, cruising, coasting, maximum deceleration [11]. Recent studies showed that three-working-phase modes are primarily used to formulate optimal train control strategies in the metro operations problems involving variable speed limit, variable track gradient and uncertainty dwelling time [12]. For example, Albrecht et al. [13,14] assumed that the traction and braking force bounded by the non-increasing speed-dependent constraints and a non-negative convex function of speed was given about the rate of energy dissipation, which found an algebraic formulation for the adjoint variables of speed in a rail line and divided into the gradient into piecewise-constant for minimizing the mechanical energy consumption by the train.

The above studies generally determined the energy-efficient speed profile by using the optimal control theory between two adjacent stations for a single train within a given running time. Different with the above continuous train control problems, some other studies tried to discrete the problem as a multiphase optimal control model incorporating complex train running conditions and practical train operations constraints including undulating track, variable speed restrictions, running resistances, speed-dependent maximum accelerating/braking forces [15]. Yang et al. [16] structured the whole metro line into many segments with the constant speed limits, and supposed the speed limit is zero at stations and at turnaround stations. This study assumed that the speed profile of trains operating on a section composes of three working phases, and analyzed the switching points about running times in mathematics between each two stations. However, most of these traditional studies considered timetable and speed profile problems separately. It may be far from the global efficiency in the whole metro system when taking the passenger demand and regenerative energy into account. The modern theory of the optimal train timetable and speed control strategies has undergone a lot of developmental refinement in recent years [17–19]. A number of recent studies have focused on this issue to reduce passenger travelling time and train energy consumption simultaneously [20,21]. For example, Ning et al. [22] proposed an operation model to optimize train headway by adjusting the train arrival times at stations. The adjustment of train arrival times was used an analytical method, and then the speed profile for each train was calculated by a suboptimal method. Yang et al. [20] developed a stochastic programming model for metro timetable optimization problem in order to jointly reduce the total traveling time and operational costs.

Most of above literatures optimized the train timetable by synchronizing the multiple trains to match the passenger demands during different working periods. It usually assumed the speed profiles as the constant parameters on sections. Energy-efficient train operations and mathematical optimization were used to develop operations strategies with better energy performance, which has been widely studied in metro systems. Energy-saving management of metro using mathematical optimization has attracted much attention in latest years [23]. For acquiring an effective and accurate mathematical model, explicit considering the real-world conditions is really important. Li and Lo [24] formulated an integrating energy-efficient operation model to jointly optimize the timetable and speed profile to reduce the net energy consumption, which firstly divided each section into several segments with constant speed limits. And for each segment inspired by the analysis of the optimal phase sequence, the authors assumed that the speed profile uses the three phases sequence (i.e., maximum acceleration, cruising, and maximum braking). However, there are two research gaps in Li and Lo [24]: (1) the cruising phase also need power force and energy consumption although the resistant is small; and (2) the authors did not consider the accelerating and braking in a section in the function of cruising time respect to the segment. The disadvantages are as follows: (1) the authors assumed that the acceleration rate was a fixed constant value, and the cruising time was calculated dependent on the segment running time, which made the speed profile discontinuousness in a section; and (2) the authors did not consider trains consumed energy during the cruising phase, and this assumption contradicted to the determination of cruising.

To address these issues, this paper develops an integrating optimization method jointly optimizing timetable and speed profiles in the whole metro line to minimize the total tractive energy consumption of all served trains. The passenger demands and multi-phase speed limits are also well considered in this paper. In particular, by using a Taylor approximation, we reformulate the problem into a strictly quadratic programming model to reduce complexity of the formulation. In order to solve the problem more efficiently, we develop an efficient algorithm combined with the ASM to obtain an approximate optimal solution within short computational time.

The main differences between some previous studies and this paper are listed as follows:

(1) Compared to literature [9], which optimized the dwell time of timetable to improve the utilization of regenerative braking energy. This paper develops an integrated timetable and speed profile optimization model to optimize both the switching-point speeds and running time for reducing the total tractive energy.
(2) Compared to literature [13,14], which designed a complex optimal train control algorithm to generate an energy-efficient speed profile by one train on a given section with the fixed running time. This paper gave a basic example can calculate the energy-efficient running time by multi-train with different working conditions on different section in a whole metro line.
(3) Compared to literature [24], which calculated the cruising time function w.r.t. segment running time on a section result in a discontinuous speed profile. This paper calculates the switching-point speeds in theoretically with consideration of