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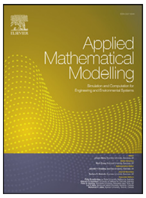

Dispersion relations of elastic waves in two-dimensional tessellated piezoelectric phononic crystals

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ABSTRACT

A type of two-dimensional tessellated piezoelectric phononic crystal is theoretically studied in this paper, formed by homogeneous piezoelectric and inhomogeneous functionally graded rectangular columns. Firstly, the propagation properties of in-plane and anti-plane Bloch waves in each piezoelectric rectangular column are systematically investigated. Furthermore, the total transfer matrices of Bloch waves are obtained based on transfer matrices of homogeneous piezoelectric and inhomogeneous functionally graded rectangular columns. Finally, the Bloch theorem is used to obtain the dispersion relations of in-plane and anti-plane Bloch waves. The influences of non-dimensional geometrical parameters and gradient profile functions on the dispersion relations are discussed based on the graphically numerical results. Under a special value of modal parameter, the dispersion surfaces and curves of in-plane Bloch waves approximatively have plane and axis symmetries respectively, and an approximate total band-gap of anti-plane Bloch waves arise. The theoretical models and numerical discussions will provide a direct guidance of multi-material 3D printing for inhomogeneous periodic structures with dispersion and band-gaps properties.

1. Introduction

Magneto-electro-elastic materials have broad application prospects in the sensor, smart material and structure engineering due to the conversional function among the magnetic energy, electric energy and mechanical energy [1–4]. The periodical structures made by piezoelectric materials, which are usually called phononic crystals (PCs), exhibit absolute band gaps characteristics of elastic waves [5–8]. This property implies a possibility of designing PC-based devices for telecommunication applications [9], spurious signals elimination [10] and engineering vibration isolation [11]. Owing to the characteristics of affecting the propagation properties of elastic waves concentrated on linear deformations, the propagation properties of Bloch waves and the characteristics of band gaps of two-dimensional (2D) piezoelectric phononic crystals (PPC) attract the investigative attentions of many researchers in recent years. The primary methods utilized to establish the theoretical models are the plane wave expansion method [12–17] monodromy-matrix method [17], the multiple scattering method [18] the transfer matrix method [18] and the finite-element method [19,20].

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Hsu and Wu [12] studied the existence of Bleustein-Gulyaev-Shimizu piezoelectric surface acoustic waves in a 2D PPC. Numerical results showed that this type of surface waves has higher acoustic wave velocities and electromechanical coupling coefficients, and produce larger band gap width than those of the Rayleigh surface waves and pseudo-surface waves. The effects of shapes and filling fraction of the rods on the existence of these gaps in relation to the physical parameters of the constituent materials were systematically discussed by Qian et al. [13] in the full band structures for 2D PPCs. It is shown that larger and more complete band gaps appear for a circular section than for a square section. This means that rods of a circular section are much better to be engineered to provide an insulation environment for high-precision mechanical systems in a given frequency range. Yang et al [14] studied the phonon-polariton behaviours of 2D PPCs. Calculation results showed that the dispersion relations can be decoupled into two independent groups: a mixed mode of the in-plane elastic waves and the electromagnetic waves and another mixed mode of the out-of-plane elastic waves and the electromagnetic waves. The elastic wave propagation in 2D magneto-electro-elastic PCs was studied by Wang et al. [15,16] with the consideration of the magneto-electro-elastic coupling. Numerical results showed that the piezoelectricity and piezomagneticity have significant effects on characteristics of the second band gap, especially for larger filling ratios. Kutsenko et al. [17] analytically formulated effective anti-plane quasistatic moduli for 2D PPCs of arbitrary anisotropy. It is observed that multiple scattering method converges much quicker to the exact moduli in comparison with the plane wave expansion as the number of Fourier coefficients increases.

Cai and Wei [18] investigated propagation characteristics of elastic waves in 2D PC consisting of parallel cylinders or cylindrical shells embedded periodically in a homogeneous host. The varying material parameters along the radial direction of the cylinders or cylindrical shells were considered. Gao et al. [19] studied the bandgap properties of a 2D PC with the two resonators embedded in a homogenous matrix. The dispersion relationship, transmission spectra, and displacement fields of the eigenmodes of this PC were systematically investigated. Zou et al. [20] discussed the propagation of elastic waves in a piezoelectric slab covered with periodically structured coatings or stubbed PC slab. Four different models and the effects of distribution forms and geometrical parameters of the structured coatings on complete band-gaps were considered, which can generate wider complete band-gaps with symmetric coatings.

During the manufacture and service of materials and structures, inhomogeneous media, such as functionally graded materials, are introduced into the architecture of composites to decrease the effects of interfacial defects and realize smooth transitions of constitutive parameters and physical properties between different materials and structures [21]. Theoretical models of the 2D PCs give the mathematical relationships of geometrical and physical parameters according to the investigations in the available references. The existence of functionally graded media will change above-mentioned parameters, and have influences on the dispersion and band-gap properties of Bloch waves [21–25]. However, the theoretical corresponding items of above-mentioned factors are infrequent existence in previous references.

Therefore, the objectives of the present research include the following aspects: (1) to establish original non-dimensional theoretical models of 2D tessellated PPCs with the consideration of geometrical architectures of orthogonality and zero-curvature, homogeneous piezoelectric and inhomogeneous functionally graded rectangular columns; (2) to calculate the dispersion surfaces, dispersion curves and band-gaps of in-plane and anti-plane Bloch waves through the transfer matrix method [18] with the consideration of different propagation properties; (3) to investigate the influences of non-dimensional geometrical, inhomogeneous parameters. The specific characteristics of the 2D tessellated PPCs are the systematical consideration of the influences of non-dimensional geometrical and gradient profile functions on the dispersion and band-gaps properties.

This paper is organized as: in Section 2, the architecture of 2D tessellated PPC is introduced, formed by homogeneous piezoelectric and inhomogeneous functionally graded rectangular columns. Propagation properties of Bloch waves and transfer matrices of homogeneous piezoelectric and inhomogeneous functionally graded primitive cells are discussed. These pave the road for the use of Bloch theorem in deriving the dispersion equations of Bloch waves for this architecture of 2D tessellated PPC—the main contents in Section 3. A non-dimensional form of the dispersion equation is derived with the accomplishment of the theoretical model of propagation of Bloch waves. Finally, in Section 4 the influences of non-dimensional geometrical parameters and gradient profile functions are discussed based on the graphically numerical results of non-dimensional dispersion surfaces, dispersion curves and band-gaps of in-plane and anti-plane Bloch waves, after which concluding remarks are made in Section 5.

2. Architecture of 2D tessellated PPC

Consider a 2D tessellated PPC formed by periodically repeating homogeneous transversely isotropic piezoelectric and inhomogeneous functionally graded rectangular columns. Four homogeneous piezoelectric rectangular primitive cells (labelled P1 and P2) and twelve inhomogeneous functionally graded rectangular primitive cells (labelled from F1 to F6) constitute the single cell of 2D tessellated PPC, shown in Fig. 1. d_{11}, d_{12}, d_{13}, d_{14}, d_{31}, d_{32}, d_{33} and d_{34} are the geometrical sizes of the rectangular primitive cells. Let the x_3-axis is the poling direction of these two piezoelectric rectangular columns. Their transversely isotropic plane is the Ox_1x_2 coordinates plane. c_{ijkl}, e_{mij}, ε_{ij} and ρ, which denote the elastic, piezoelectric, dielectric and mass density parameters respectively, are the constitutive parameters of P1 and P2 transversely isotropic piezoelectric rectangular columns. The constitutive parameters are labeled with the superscript “$'$” and “$''$” respectively. In latter formulation, if the physical parameter does not have any superscript, it will be appropriate for these two piezoelectric rectangular columns.