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Uncertain random assignment problem

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ABSTRACT

This paper proposes an uncertain random assignment problem in which random variables coexist with uncertain variables. Critical values of uncertain random variables are used to rank uncertain random variables. An uncertain random simulation algorithm is developed in order to obtain chance distributions and critical values of uncertain random variables. An α-optimistic model is presented. A combined optimization approach is designed to solve the α-optimistic model. This approach incorporates uncertain random simulation into branch and bound algorithm. Finally, an example application of the approach is presented.

1. Introduction

The Classic Assignment Problem is one of the fundamental problems in the area of combinatorial optimization. Easterfield [1] first studied the algorithm for the Classic Assignment Problem. Kuhn [2] proposed the famous Hungarian method in order to provide the solution of classic assignment problem. Kurtzberg [3] developed approximation methods for large scale assignment problems. Other scholars have discussed the semi-assignment problem [4], the minimum deviation assignment problem [5], the fractional assignment problem [6], the k-cardinality assignment problem [7], the multidimensional assignment problem [8–10], the generalized assignment problem [11] and the priority based assignment problem [12].

All of the above studies assume that coefficients are constants. But, in many applications, coefficients are not constants but random variables. Thus, researchers study random assignment problems [13], [14]. Mézard and Parisi [15] use the replica method to study the expected optimal value. Lazarus [16] develops the lower bound on the value of a random assignment problem. Buck et al. [17] conjecture an explicit formula for the expected value of the random k-assignment. Parviainen [18] discusses an assignment problem with discrete random costs. Li et al. [19] present a new genetic algorithm selection scheme to solve the random assignment problem. Sethuraman and Ye [20] consider the random assignment problem on a uniform preference domain. Pour et al. [21] study a new stochastic personnel assignment problem.

However, when a worker is assigned a new job, we often do not have statistical data available in order to assess his or her performance. In such a case, it is impossible to obtain the probability distributions of coefficients. However, we can invite experts to estimate coefficients based on their knowledge and experience. Such an estimation has an element of human uncertainty, as it is usually to provide an estimation. In order to deal with this human uncertainty, Liu [22] proposes an uncertain variable and establishes uncertainty theory. Thereafter, some researchers apply the uncertainty theory in order to study various uncertain problems. Yang and Ralescu [23] propose a new Adams method for solving uncertain differential

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Table 1
Assumption and mathematical model for assignment problem.

	Assumption(coefficient)	Mathematical model
Deterministic model [1–12]	Constant	Integer programming
Random model [13–21]	Independent random variable	Stochastic programming
Uncertain model [27]	Independent uncertain variable	Uncertain programming
Our model	Independent uncertain variable and independent random variable	Uncertain random programming

equations. Hosseini and Wadbro [24] have conducted an analysis of reliability and stability in uncertain networks. Some scholars have carried out research on uncertain programming problems. For example, Dalman [25] considers an uncertain multi-objective multi-item solid transportation problem. Veresnikov et al. [26] designs an uncertain technical systems. In particular, Zhang and Peng [27] study an uncertain assignment problem in which coefficients are uncertain variables.

However, randomness and human uncertainty may coexist in complex systems. For example, when we sell new products and existing products to the existing markets, we can obtain the probability distribution of the demand for existing products from historical data. But we cannot obtain the probability distribution of the demand of new products due to a lack of data. In such situations, we can invite experienced experts to estimate the demand for new products. Therefore, randomness and human uncertainty simultaneously appear in such a problem. Liu [28] proposes the use of chance theory to deal with uncertain random problems. Qin [29] then solves a portfolio selection problem with random and uncertain returns. Gao and Yao [30] discuss the uncertain random stationary increment process and the uncertain random renewal process.

Scholars first study deterministic models [1–12], then study models with random variables [13–21]. They then study models with uncertain variables [27]. It is therefore natural to think about how to solve the assignment problem where uncertain variables coexist with random variables. We present an uncertain random assignment model under the assumption that coefficients are independent uncertain variables and independent random variables. Since constant, random variable and uncertain variable are special cases of uncertain random variables, previous models for the assignment problem are special cases of our model. The different types of model are listed in Table 1:

Since uncertain random variables may not be differentiable and continuous, usual optimization methods cannot be used to solve uncertain random models. Researchers use simulation in order to obtain numerical solutions. Ke et al. [31] design an uncertain random simulation for solving uncertain random project scheduling problems. However, their uncertain random simulation is likely to yield different results every time. Sheng and Gao [32] propose a special algorithm for uncertain random network optimization.

In this paper, we present a new simulation algorithm. Our algorithm generates sample points uniformly, while the algorithm in [31] generates sample points stochastically. In fact, it is difficult to generate sample points “stochastically” for complex uncertain random variables. The algorithm in [31] is unsteady. Algorithms in [31,32] are specially designed for project scheduling problems and network optimization problems. They can be used to simulate the chance distributions of increasing functions. However, our algorithm is general and without any restriction on functions. Our algorithm can be used to simulate chance distributions of increasing functions and decreasing functions.

The remainder of this paper is organized as follows: In Section 2, some basic concepts and theorems of uncertainty theory and chance theory are introduced. In Section 3, we propose ranking criteria for the evaluation of uncertain random variables, and design an uncertain random simulation algorithm in order to obtain the chance distribution and critical values. In Section 4, we propose an uncertain random assignment problem and present the α-optimistic model. In order to solve the corresponding optimization problem, a branch and bound algorithm integrated with uncertain random simulation is developed. Several conclusions are drawn in Section 5.

2. Preliminary

In this section, we introduce some concepts and theorems on uncertain theory and chance theory.

2.1. Uncertainty theory

Let Γ be a nonempty set and let $\mathcal{L}$ be a σ-algebra over Γ. A set function $\mathcal{M} : \mathcal{L} \to [0, 1]$ is called an uncertain measure if it satisfies the following four normality axiom, duality axiom, subadditivity axiom and product axiom [22], [33].

Definition 1 (Liu [33]). The uncertain variables $\xi_1, \xi_2, \ldots, \xi_n$ are said to be independent if

$$\mathcal{M}\left\{\bigcap_{i=1}^{n}\{\xi_i \in B_i\}\right\} = \bigwedge_{i=1}^{n}\mathcal{M}\{\xi_i \in B_i\}$$

for any Borel sets $B_1, B_2, \ldots, B_n$.