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Complex analytical solutions for flow in hydraulically fractured hydrocarbon reservoirs with and without natural fractures

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ABSTRACT

Reservoir drainage towards producer wells in a hydraulically and naturally fractured reservoir is visualized by using an analytical streamline simulator that plots streamlines, time-of-flight contours and drainage contours based on complex potentials. A new analytical expression is derived to model the flow through natural fractures with enhanced hydraulic conductivity. Synthetic examples show that in an otherwise homogeneous reservoir even a small number of natural fractures may severely affect streamline patterns and distort the drainage contours. Multiple parallel natural fractures result in a drainage region that is narrower in the direction normal to the natural fractures while the drainage reach is larger in the natural fracture direction. Reservoirs with numerous natural fractures are shown to be characterized by more tortuous drainage patterns than reservoirs without natural fractures. Finally, the analytical flow model for naturally fractured reservoirs is applied to a natural analog of flow into hydraulic fractures. The tendency of the injected fluid to stay confined to the fracture network as opposed to matrix flow is entirely controlled by the hydraulic conductivity contrast between the fracture network and the matrix.

1. Introduction

In the hydrocarbon industry, economic development of shale reservoirs with low permeability is commonly achieved by hydraulically fracturing such reservoirs. When natural fractures are present in the reservoir they may support the flow of reservoir fluids into to the producer wellbore. Maximizing a producer well's performance therefore requires optimal hydraulic fracture planning [1–3], which in turn necessitates adequate flow models for hydraulic and natural fractures. In this study, we visualize fluid drainage by hydraulic and natural fractures by employing analytical methods, in contrast to various semi-analytical and numerical solution methods commonly used [4–6].

Potential flow theory, which provides closed-form analytical solutions of the Laplace equation, is at the heart of this study and is applied to derive models of hydraulic and natural fractures for flow simulation. Hydraulic fractures in reservoirs are directly connected to a producer wellbore and therefore aid in draining reservoir fluids. Such fractures were already modeled analytically in an earlier study [7]. Natural fractures, on the other hand, are conduits or cracks that either expedite or impede the flow of reservoir fluids. When such fractures are mineralized they may form impermeable barriers inside the reservoir,

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which has been modeled in one of our prior studies [8]. However, the modeling of flow acceleration through multiple cracks with enhanced hydraulic conductivity has not been modeled before by any analytical method.

The main purpose of our present study is to derive an efficient 2D analytical description for permeable fractures to enable rapid modeling of flow diversion in reservoirs, made up of a porous medium containing multiple natural fracture systems. In potential theory one can obtain new solutions by superposing existing solutions. A well-known example of superposition in fluid mechanics, aerodynamics and electromagnetism is the singularity doublet (or point dipole), which is obtained by superposing a point source and a point sink in a limiting process [9–11]. With a different approach one can transform an infinite amount of point sinks into an interval sink [12,13], which is how we previously modeled a hydraulic fracture [14]. Similarly, an infinite number of singularity doublets can be transformed into a line doublet [8], which can approximate high and low conductivity zones [15]. In this paper, we present a new analytical model of a natural fracture, which we created by superposing an infinite amount of line doublets.

Although solutions from potential flow theory are obtained only after certain restricting assumptions (discussed in Section 2), the fact that the solutions are closed-form formulae implies that the computational cost of visualizing these solutions is low. Consequently, densely clustered streamline tracking is achievable at low costs, while enabling uniquely accurate tracking of the time-of-flight-contours (TOFCs) and drainage contours of reservoir fluids. Another benefit of the analytical solutions is that velocity fields and pressure fields are obtainable at high resolution. The accuracy of the analytical models has been verified by matching results to those of a numerical simulator [14].

This paper is structured as follows. Basic assumptions of fluid flow, a discussion of hydraulic and natural fracture models, as well as a description of our visualization method and its benefits can be found in Section 2. Section 3 is dedicated to basic illustrations of the analytical elements presented in this study. In Section 4, we combine analytical natural and hydraulic fracture elements to visualize the drainage of a hydrocarbon reservoir. Lastly, in Section 5, we illustrate an application of our analytical natural fracture element by modeling the flow through a complex natural fracture network from a polished rock slab. The fracture network in the slab shows fluid injection paths and particle flow through the adjacent matrix. Applying the flow reversal principle, the same slab may serve as an analog for drainage by a natural fracture network in a subsurface reservoir.

2. Methodology

This section addresses basic assumptions on the reservoir and the fluids inside it, and is followed by a brief review of previously developed hydraulic and natural fracture models. Next, we explain our visualization method, based on potential theory for fluid flow [16,17] and the analytical element method [18–26]. Lastly, we highlight the benefits of analytical models.

2.1. Fluid flow model assumptions

Potential theory and conformal mappings [27] have been applied in various fields to model idealized flow in a viscous continuum [9–11,28] and form the foundation for each analytical element in this paper. Previous studies have advocated the modeling of Darcy flow in porous media by potential methods [25,29–35]. The main assumptions behind modeling Darcy flow with potential theory concern both the reservoir and the reservoir fluids.

The reservoir model in our study is comprised of a porous matrix assumed to be homogeneous, incompressible and confined to a relatively thin layer with large lateral extent. The layer is considered to be a part of a larger reservoir whose boundaries lie far away from the flow region studied so that boundary effects can be neglected. The assumption of a reservoir comprised of a homogeneous matrix implies constant reservoir porosity (the percentage of the reservoir pore space available for fluid flow) and permeability (the ease with which fluids can traverse through the reservoir). The only heterogeneities in our reservoir model are the fractures, which may enhance fluid flow locally. Vertical pressure gradients such as gravity are neglected, justified by our assumption of a relatively thin reservoir space. With these reservoir assumptions we can limit the reservoir model to two-dimensional fluid flow, even though fluid flow in three dimensions can also be described with the analytical element method [19,26].

The fluid flow we consider is irrotational, incompressible, immiscible and isothermal. Consequently, the fluids' viscosity and density are constant. We also assume negligible capillary pressure and disregard any wettability and relative permeability effects of the reservoir fluids. The initial pressure of the reservoir, P_0, will after flow disturbance by a change agent $\Delta P(z)$, become:

$$P(z) = P_0 + \Delta P(z). \qquad [\text{Pa}] \tag{1a}$$

When the reservoir fluid develops a pressure gradient, $P(z)$ is mainly governed by the reservoir properties (porosity, permeability, and height) and reservoir fluid property (viscosity). Assuming positive strength for injectors and negative strength for producers (see also Appendix A.1), we can express $P(z)$ as

$$\Delta P(z) = -\frac{\phi(z)\mu}{k}. \qquad [\text{Pa}] \tag{1b}$$