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A unified method and its application to brake instability analysis involving different types of epistemic uncertainties

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ABSTRACT

The key idea of the proposed method is the use of the equivalent variables named as evidence-based fuzzy variables, which are special evidence variables with fuzzy focal elements. On the basis of the equivalent variables, an uncertainty quantification model is established, in which the unified probabilistic information related to the uncertain responses of engineering systems can be computed with the aid of the fuzziness discretization and reconstruction, the belief and plausibility measures analysis, and the interval response analysis. Monte Carlo simulation is presented as a reference method to validate the accuracy of the proposed method. The proposed method then is extended to perform squeal instability analysis involving different types of epistemic uncertainties. To illustrate the feasibility and effectiveness of the proposed method, seven numerical examples of disc brake instability analysis involving different epistemic uncertainties are provided and analyzed. By conducting appropriate comparisons with reference results, the high accuracy and efficiency of the proposed method on quantifying the effects of different epistemic uncertainties on brake instability are demonstrated.

1. Introduction

The system instability of automotive brakes can cause irritating noise and vibrations. The pollution of brake squeal noise is a major concern in industry, and the noise usually occurs in the frequency range between 1 kHz and 16 kHz [1]. Due to consistent customer complaints and high warranty costs, brake squeal issue has received a large amount of attention from researchers.

It has been found that brake instability is significantly affected by the geometric dimensions, boundary conditions, material properties or external loads of brake systems. Traditional instability analysis of brake systems is usually carried out in the context of deterministic parameters, such as the work of Júnior et al. [2]. However, uncertainty is inherent and unavoidable in almost all engineering structures and mechanical systems. In the automotive brake systems, uncertainties associated with material properties, geometric dimensions, subjective experiences, boundary conditions are ubiquitous. As a comparatively new research field, the uncertainty analysis of brake instability is attracting more and more attention of analysts in recent years. Generally, the uncertainty can be classified into aleatory and epistemic types [3].

Aleatory uncertainty, which is known as objective or stochastic uncertainty, is related to inherent variability of a physical system. Aleatory uncertainty is always represented by probability theory with random variables or random processes. Brake

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instability analysis under aleatory uncertainty has achieved great attention in the last few years. To quantify the aleatory uncertainty in brake instability analysis, a number of probabilistic approaches based on piecewise polynomial chaos expansion [4], Monte Carlo simulation and kriging surrogate [5,6], or perturbation technique [7] have been developed. These probabilistic approaches have achieved significant successes, but precise probability distribution on each uncertain parameter has to be available in these approaches. However, in most cases, the substantial information to construct precise probability distributions of uncertain parameters may be insufficient. Therefore, the traditional probabilistic approaches are inappropriate to represent imprecise uncertainty and it is desired to develop some alternative methods, such as non-probabilistic approaches.

Epistemic uncertainty, which derives from lack of knowledge or incomplete information, is another type of uncertainty. Many non-probabilistic theories have been developed to quantify epistemic uncertainty, such as interval analysis [8], fuzzy sets [9], evidence theory [10] and probability box (p-box) [11]. Epistemic uncertainty analyses have been widely conducted in varieties of engineering fields, including the acoustic response analysis [12], heat conduction analysis [13] and structural reliability and safety analysis [14,15] and so on. And recently, the epistemic uncertainty quantification of brake squeal instability has undergone a rapid development. Based on interval analysis, Lü and Yu [16] made an exploration on the system stability optimization of an uncertain disc brake with limited data. To quantify the fuzzy uncertainty of unstable frequencies, Gauger et al. [17] proposed a fuzzy arithmetical approach for the instability analysis of an uncertain disc brake. And Giannini [18] developed a fuzzy model to characterize the instability tendency of a laboratory brake with fuzzy parameters. By integrating fuzzy data into a friction induced vibration system, Massa et al. [19] presented a complete strategy to perform a fuzzy study to analyze the effects of fuzzy uncertainty on system instability. In order to quantify the uncertainty with incomplete or conflict information, Lü et al. [20] developed an imprecise probability approach for brake squeal instability analysis based on evidence theory.

From the above-mentioned work, it can be seen that some inspiring progresses have been made on the epistemic uncertainty quantification of brake squeal instability. Nevertheless, some important issues still remain unsolved in this research field. For example, recently, there has been a great interest in developing effective methods for handling the engineering problems with mixed uncertainties [21], where different types of uncertain variables simultaneously appear in the same issue. However, most of the existing brake squeal instability analyses under epistemic uncertainty are carried out just considering single type of uncertainty (e.g., interval uncertainty, fuzzy uncertainty or evidence uncertainty). The researches considering multiple types of epistemic uncertainties simultaneously have been rarely investigated. For another example, in the previous researches, the different squeal instability analyses under epistemic uncertainty are carried out separately and based on entirely different models, rather than implemented in a unified framework. The analysis methods which can be applied to brake instability analysis under different epistemic uncertainties are still unexplored.

Therefore, this paper aims at developing an effective unified method for epistemic uncertainty analysis and extending it to brake instability analysis involving different types of epistemic uncertainties. The proposed method can be not only applied to the brake instability analysis under single type of epistemic uncertainty, but also suitable for the analysis under multiple types of epistemic uncertainties. The remainder of this paper is organized as follows. Some basic approaches for modeling epistemic uncertainty are firstly introduced in Section 2. Then, a unified analysis method for epistemic uncertainty analysis is derived in Section 3. The proposed method is extended to conduct brake squeal instability analysis in Section 4. Finally, seven numerical examples and some discussions are provided in Section 5. Conclusions and remarks about this study are given at the end of this paper.

2. Approaches for modeling epistemic uncertainties

2.1. Interval approach

In interval approach, the epistemic uncertainties are modeled as interval variables with assigned bounds. Let $\mathbf{B}^I = \{B_1^I, B_2^I, \ldots, B_s^I\}^T$ denotes an interval vector composed by s independent variables. $\mathbf{B}^I$ can be expressed as

$$\mathbf{B}^I = [\underline{\mathbf{B}}, \bar{\mathbf{B}}] = \mathbf{B}^C + \Delta\mathbf{B}^I, \quad \Delta\mathbf{B}^I = [-\Delta\mathbf{B}, \Delta\mathbf{B}], \quad \Delta\mathbf{B} = \frac{\bar{\mathbf{B}} - \underline{\mathbf{B}}}{2}, \quad \mathbf{B}^C = \frac{\underline{\mathbf{B}} + \bar{\mathbf{B}}}{2}$$

where $\underline{\mathbf{B}}$ and $\bar{\mathbf{B}}$ represent the lower and upper bounds of $\mathbf{B}^I$, respectively; $\Delta\mathbf{B}$ and $\mathbf{B}^C$ represent the maximum deviation and the midpoint of $\mathbf{B}^I$, respectively; $\Delta\mathbf{B}^I$ denotes the deviation interval of $\mathbf{B}^I$. $\mathbf{B}^I$ can also be expressed in component forms

$$B_i^I = [\underline{B}_i, \bar{B}_i] = B_i^C + \Delta B_i^I, \quad \Delta B_i^I = [-\Delta B_i, \Delta B_i],$$
$$\Delta B_i = \frac{\bar{B}_i - \underline{B}_i}{2}, \quad B_i^C = \frac{\underline{B}_i + \bar{B}_i}{2}, \quad i = 1, 2, \ldots, s$$

Let $y = f(\mathbf{X})$ denotes the response function of an uncertain system and $\mathbf{X}$ is the vector of system parameters. For the system with only interval variables $\mathbf{B}^I$, the system response can be expressed as $y = f(\mathbf{B}^I)$. Here, y becomes an interval response due to the influence of $\mathbf{B}^I$. A number of available methods have been developed to estimate the interval response, such as the vertex method [22] and interval perturbation method [23].