Applied Mathematical Modelling 56 (2018) 186–201

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Reliability-based user equilibrium in a transport network under the effects of speed limits and supply uncertainty

Anny B. Wang[a,b], W.Y. Szeto[c,d,*]

[a] Guangzhou Municipal Engineering Design & Research Institute, Guangdong, PR China
[b] School of Civil Engineering and Transportation, South China University of Technology, Guangdong, PR China
[c] Department of Civil Engineering, The University of Hong Kong, Hong Kong, PR China
[d] The University of Hong Kong Shenzhen Institute of Research and Innovation, Shenzhen, PR China

ARTICLE INFO

Article history:
Received 22 February 2017
Revised 6 November 2017
Accepted 20 November 2017

Keywords:
Speed limit
Supply uncertainty
Distribution-free reliability-based user equilibrium
Inefficiency

ABSTRACT

This study investigates the system-wide traffic flow re-allocation effect of speed limits in uncertain environments. Previous studies have only considered link capacity degradation, which is only one of the factors that lead to supply uncertainty. This study examines how imposing speed limits reallocates the traffic flows in a situation of general supply uncertainty with risk-averse travelers. The effects of imposing a link-specific speed limit on link driving speed and travel time are analyzed, given the link travel time distribution before imposing the speed limit. The expected travel time and travel time standard deviation of a link with a speed limit are derived from the link travel time distribution and are both continuous, monotone, and convex functions in terms of link flow. A distribution-free, reliability-based user equilibrium with speed limits is established, in which travelers are assumed to choose routes that minimize their own travel time budget. A variational inequality formulation for the equilibrium problem is proposed and the solution properties are provided. In this study, the inefficiency of a reliability-based user equilibrium flow pattern with speed limits is defined and found to be bounded above when supply uncertainty refers to capacity degradation. The upper bound depends on the level of risk aversion of travelers, a ratio related to the design and worst-case link capacities, and the highest power of all link performance functions.

1. Introduction

High-speed driving can increase the number of crashes on a road and increase the severity of injuries in a crash (see [1,2]). A speed limit is commonly imposed on a road to restrict the maximum allowable driving speed and hence prevent traffic accidents. An example in practice is the Variable Speed Limit system used in Europe and the United States (see [3,4]), which is designed to reduce the speed difference (harmonize the traffic flow) on hazardous highway segments and thus decrease the rear-end collision rate. In addition to improving road safety, speed limits also reduce high-speed-related fuel consumption and emission problems (see [5,6]).

Recent studies of optimal speed limit design (e.g., [7,8]) have mainly focused on the local effects of speed limits (i.e., on a single link). McKnight and Klein [9] and Grabowski and Morrisey [10] recognized the potential traffic flow re-allocation

* Corresponding author at: Department of Civil Engineering, The University of Hong Kong, Hong Kong, PR China.
E-mail address: ceszeto@hku.hk (W.Y. Szeto).

https://doi.org/10.1016/j.apm.2017.11.019
0307-904X/

effect of speed limits, and Yang et al. [11] and Wang [12] investigated how imposing speed limits reallocates traffic flows in an equilibrium manner at a macroscopic network level. They suggested that imposing a speed limit alters the link travel time–flow relationship, but a traffic assignment principle still applies. Yang et al. [13] then proposed a bi-level variable speed limit design model in which a user equilibrium (UE) model with speed limits was incorporated as the lower-level model. Yang et al. [14] considered the non-obedient behavior of travelers when confronted with speed limits. These studies assume that the link travel times are deterministic. Yan et al. [15] examined the effects of speed limits on traffic equilibrium when the link travel times are random due to link capacity degradations, prompting further investigation into reliability-based user equilibrium (RUE) problems that consider travel time uncertainty and the effects of speed limits.

Travel time uncertainty is mainly caused by demand (flow) uncertainty and network supply uncertainty. The former is due to the demand fluctuation caused by, e.g., temporal factors (e.g., time of day, day of the week, or seasonal effect) and special events [16]. Relevant studies include [17,18]. Supply uncertainty (e.g., [19,20]) mainly involves link capacity variation and degradation due to factors such as adverse weather conditions, road maintenance, and traffic accidents, etc. Szeto and Wang [21] suggested that variations in the link free flow travel times due to adverse weather conditions are also a type of supply uncertainty. If travel time uncertainty is taken into consideration, the effects of imposing a link-specific speed limit on the link driving speed and travel time are more difficult to analyze, but they have practical importance for accurate evaluation of the benefits of imposing speed limits.

Travel time uncertainty also affects travelers' route choices as pointed out by the studies [22–26]. Several approaches have been proposed to model the travelers' selfish routing behavior, such as the stochastic dominance approach [27,28] based on an axiomatic model of risk-averse preferences. Travelers are assumed to possess a utility function and choose routes that maximize their own utility [27,28]. Another approach [29] assumes that travelers choose routes to maximize their own probability of on-time arrival. The most common are the mean-risk type approaches, in which travelers are assumed to choose routes according to the expected travel time and the variability of travel time, and then make a tradeoff between these two factors [30,31]. Lo et al. [20] proposed the concept of travel time budget (TTB), which is the sum of the expected travel time and the travel time standard variation multiplied by a scalar. At equilibrium, travelers choose routes to minimize their own travel time budget, which is associated with a predefined on-time arrival probability [20]. Nie and Wu [32] and Nie [33] proposed the concept of percentile travel time (PTT) for a desired on-time arrival probability, and travelers are assumed to minimize their own PPT. Chen and Zhou [34] proposed the concept of mean and excess travel time (METT), which is the sum of the travel time budget and an excess travel time. At equilibrium, travelers choose routes to minimize their METT [34]. Comparisons among the mean-risk type approaches are provided by [35], and the TTB approach is most widely adopted by scholars [36,37]. Route choices of travelers also affect the distribution of traffic flows. Therefore, it is important to consider travelers' selfish routing behavior in the speed limit analysis to accurately assess the benefits of imposing speed limits.

This study focuses on the effects of speed limits on link travel times and driving speeds in an environment with supply uncertainty, given the link travel time distributions before the speed limits are imposed. The mean link travel time and the link travel time standard deviation of a link with a speed limit is derived from the travel time distribution. This study extends the concept of TTB to consider the effects of speed limits and proposes a distribution-free RUE model. A variational inequality (VI) formulation for the proposed model is given, and the solution properties are examined. The inefficiency of an RUE flow pattern with speed limits is also defined and analyzed. The main differences between our study and [15] are as follows.

- Supply uncertainty is considered to include more than capacity degradation.
- Link capacity is not required to follow a uniform distribution. In fact, the proposed approach only requires that the probability density/mass function of link travel time must be known. This approach does not exclude a case in which the link travel time distribution is derived from a capacity distribution.
- The RUE model established here does not require knowledge of the path travel time distributions (i.e., it is distribution-free), and it captures the travel time correlations between links.
- Rigorous analyses of the *mathematical* properties of the mean and standard deviation of link travel times and the solution properties of the RUE model are conducted.
- The *inefficiency* of an RUE flow pattern is defined and examined.

The study provides the following contributions.

- An analysis of the effects of a speed limit on the random link travel time and driving speed given the link travel time distribution before the speed limit is imposed, which provides fundamental computation formulae for the mean link travel time and link travel time standard deviation after a speed limit is imposed.
- A distribution-free RUE model to capture both risk aversion and selfish routing behavior of travelers in a transportation network with speed limits, and consideration of the travel time correlations between links.
- An examination of the analytical and counterintuitive properties of the RUE flow pattern with speed limits.
- A definition and investigation of the inefficiency of an RUE flow pattern with speed limits.

The remainder of this paper consists of four sections. In Section 2, how the distributions of random driving speed and travel time on a link are affected by the imposition of a link speed limit is analyzed, and the mean link travel time and link travel time standard deviation are derived. In Section 3, an RUE model for a transportation network with speed limits