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An extended thermal-medium crack model

Xian-Ci Zhong*, Xiao-Yu Long, Li-Hua Zhang

School of Mathematics and Information Science, Guangxi University, Nanning, Guangxi 530004, PR China

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ABSTRACT

It is important to investigate the effects of heat conduction of crack interior on thermoelastic fields of a cracked material. In this paper, an extended thermal-medium crack model is proposed to address the influences of the thermal conductivity inside an opening crack on the induced thermoelastic fields. Then the problem of a penny-shaped crack in a transversely isotropic material is investigated under applied mechanical and uniform heat flow loadings. Based on the Hankel transform technique, the governing partial differential equations are transformed to ordinary differential equations, then to a system of coupled dual integral equations. The thermoelastic fields around the penny-shaped crack are obtained explicitly by solving the derived dual integral equations. Numerical results are reported to show the influences of the thermal conductivity of crack interior on partial insulation coefficient, temperature change across crack and thermal stress intensity factor. As compared to the known thermal-medium crack model, the proposed one exhibits more applicability.

1. Introduction

Due to the thermal-stress effects, the micro-crack and failure behavior of engineering structures induced by applied severe thermal loadings may be created [1,2]. In order to understand the reliability and service life of engineering structures, it is significant to investigate the thermoelastic fields of a cracked material in modelling some real-world phenomena, even to make a highly idealized steady-state thermal analysis [3,4]. One can see that a great number of works have been reported to address the thermoelastic fields of a cracked material under applied thermal loadings since the works published about fifty years ago [5,6].

Here, we specially focus on the thermoelastic analyzes for three dimensional (3D) cracks in an elastic material under thermal loadings according to the theory of fracture mechanics. By prescribing the temperature at the surface of the crack, the problem of a penny-shaped crack in an isotropic material was dealt with by Olesiak and Sneddon [5], where the Hankel transform technique was used. Under the loading of uniform heat flow, the closed-form solutions were obtained by Florence and Goodier [7] for the thermoelastic fields around the penny-shaped crack in an infinite solid. Kassir and Sih [8] considered the problem of a flat elliptical crack in a solid subjected to the uniform temperature or/and temperature gradients. The thermal stress distributions induced by a penny-shaped crack in an elastic slab were given in [9], where various temperature and stress loadings prescribed on the faces of the crack and the slab were equipped. The thermal stresses were further analyzed by considering a penny-shaped crack embedded in a semi-infinite elastic solid in [10] and lying at the interface of bi-materials in [11,12], respectively. Recently, an arbitrarily shaped three-dimensional crack in an isotropic material and at the interface of bi-materials were investigated by using the displacement discontinuity method and singular integral

* Corresponding author.
E-mail addresses: zhongxianci@163.com, xczhong@gxu.edu.cn (X.-C. Zhong).

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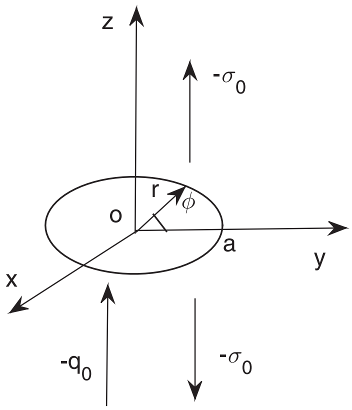

Fig. 1. A penny-shaped crack embedded in a transversely isotropic solid with uniform mechanical and heat flux loadings.

equation methods [13–15]. Moreover, when the material is transversely isotropic [16], the thermoelastic fields weakened by a crack have attracted much attentions. For example, a series of three-dimensional crack problems for transversely isotropic solids were addressed by considering static and dynamic thermal loadings [17–22]. Chen et al. [23] extended the potential theory method to give an exact fundamental solution for a penny-shaped crack in a transversely isotropic elastic solid subjected to a point-temperature loading. The potential theory method was further used to deal with the problems of a penny-shaped crack in an elastic solid with various thermal loadings in [24,25], respectively.

In the above mentioned works, the thermally insulated crack model is mainly used and the thermal stresses induced by applying thermal loadings are focused on comprehensively. However, in fact, the thermal conductivity of a medium is not fully insulated and it is partially insulated. In particular, when a crack is opened under applied mechanical loadings, a medium such as air is filled with the crack interior. Under the consideration of the partially insulated crack, various crack-face boundary value conditions have been proposed and used to study thermoelastic crack problems [26–30]. One can see that the thermal-medium crack model proposed by Zhong and Lee [29] can be used to study the effects of applied mechanical loadings and the thermal conductivity of crack interior on the thermal stress intensity factor. It has been applied to investigate the problem of a penny-shaped crack in an infinite isotropic material [31]. In the present paper, we further extend the thermal-medium model to investigate the problem of a penny-shaped crack in a transversely isotropic solid under a combination of mechanical and uniform heat flow loadings. The Hankel transform method is applied to obtain the thermoealstic fields explicitly, and to analyze the effects of heat conduction of crack interior on physical quantities of concerns. The paper is organized as follows. In Section 2, the basic equations for transversely isotropic thermoelastic material are given. The crack-face boundary conditions are discussed and an extended thermal-medium crack model is proposed. Section 3 gives the explicit solutions of the thermoelastic fields around the penny-shaped crack. In Section 4, numerical results are reported to address the effects of thermal conductivity inside the penny-shaped crack on the physical parameters such as the thermal stress intensity factor and the jump of temperature change across crack. The main conclusions are covered in Section 5.

2. Problem formulation

Considering that a penny-shaped crack is embedded in a transversely isotropic thermoelastic solid, which is subjected to a simple uniform tensile loading and a uniform heat flux. Fig. 1 is depicted to show the geometry of the penny-shaped crack in a transversely isotropic solid and the loadings of uniform stress and heat flux. The crack is located at the circular region in the *xoy*-plane with $0 \le r \le a$ and $z = 0,$ where (r, ϕ, z) is a system of cylindrical polar coordinates. The mechanical loading σ_0 is assumed to be applied at the crack faces and q_0 is the uniform heat flux.

With regard to the temperature change $\theta(r, z)$ under the steady state, the governing partial differential equation is expressed as

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \theta}{\partial r}\right) + k^2\frac{\partial^2 \theta}{\partial z^2} = 0, \tag{1}$$

where $k^2 = k_z/k_r,$ k_r is the coefficient of the thermal conductivity in the planes of isotropy and k_z is the coefficient of thermal conductivity in the *z*-direction. The nonvanishing heat fluxes q_r and q_z can be calculated as

$$q_r = -k_r\frac{\partial \theta}{\partial r}, \qquad q_z = -k_z\frac{\partial \theta}{\partial z}. \tag{2}$$