Applied Mathematical Modelling 57 (2018) 376–390

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Applicable symbolic computations on dynamics of small-amplitude long waves and Davey–Stewartson equations in finite water depth

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A R T I C L E I N F O

Article history:
Received 6 April 2017
Revised 20 December 2017
Accepted 8 January 2018

Keywords:
Davey–Stewartson equations
Solitary waves
Painlevé analysis
Hamiltonian approach
The (G'/G)-expansion method
Stability

A B S T R A C T

Some classes of nonlinear partial differential equations can be reduced to more tractable single nonlinear equations via the lowest order of the perturbed reductive technique. The nonlinear and dispersive waves of the shallow-water model are investigated throughout a finite depth of fluid under the influence of surface tension and gravitational force in an attempt to derive the Davey–Stewartson equations (DSEs). Dispersion properties of the model and conservation laws of the DSEs are studied. We apply the Painlevé analysis to investigate the integrability of the DSEs and to construct the Bäcklund transformation via the truncation Painlevé expansion. By employing the Bäcklund transformation, the Hamiltonian approach and the (G'/G)-expansion method to the DSEs, new traveling solitary and kink wave solutions are obtained. It is revealed that the amplitudes of waves decrease with increasing Ursell parameter. The trend of the wave profile does not change with time. In addition, through the Hamiltonian approach, it is found that the amplitude of the waves increases with increasing energy constant. Furthermore, the phase portrait method is applied to the resulting nonlinear first-order differential equations of the DS model to reveal its stability.

1. Introduction

Wave propagation in incompressible and inviscid fluids is a classic problem in mathematical physics and engineering. Particularly, surface waves have been intensively studied and many model equations were presented in shallow and deep water, and the study of shallow water theory is more widely known [1–15]. Nonlinear wave propagation has been studied in many fields such as plasma physics, fluid mechanics, quantum field theory, convective heat transfer and solid-state [16–20]. Much attention has been focused on the integrability of the nonlinear partial differential equations (PDEs) that describe

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https://doi.org/10.1016/j.apm.2018.01.017

these models. It was found that there was a close relation between the integrability of these equations and Painlevé property which can be investigated via different methods, such as the Ablowitz–Ramani–Segur algorithm (ARS) and the Weiss–Tabor–Carnevale (WTC) method [21–25]. Recently, different methods for finding soliton solutions, kink, cnoidal waves, and other waves of these PDEs have been suggested, developed and extended. These methods become more attractive due to their advantages in explaining the physical mechanisms [26–34].

The Korteweg–de Vries (KdV) and the nonlinear Schrödinger (NLS) equations are widely applied approximations for many problems in mathematics and physics. One of these problems is the nonlinear shallow water wave problem. It was widely studied by many authors to derive single nonlinear PDEs through different asymptotic expansion methods (e.g. Dullin et al. [5,6], Lizuka and Wadati [2], and Abourabia et al. [7,9]). The first two groups studied the derivation of the Camassa–Holm, NLS, and diffusion equations in one-dimension without finding solutions, while Abourabia et al. derived and solved the KdV and NLS equations to describe the traveling solitary waves in their models. The same problem without including the effect of surface tension was revisited by Demiray [10], and a set of KdV equations was obtained using the multiple scaling method. In addition, the nonlinear water wave model without the effect of surface tension was investigated numerically [13,14] and analytically [11,12], and solitary wave solutions were obtained. In this study, we deal with the Davey–Stewartson equations (DSEs) that focus on the long-wave and short-wave resonances and other patterns of the wave propagation.

The Davey–Stewartson equations were initially formulated by Davey and Stewartson for the evolution of weakly nonlinear water waves under the gravity force effect only [35]. The interaction between long and short waves in shallow water was investigated by Djordjevic and Redecopp [36] and Benney and Roskes [37]. The localized soliton and dromion solutions, i.e. solutions decaying in all directions and preserving their form after collisions with others solitons, of the DSEs have been obtained by Boiti et al. [38] using the inverse scattering method. In addition, the DSEs have been studied by Selima et al. [34] for the gravity waves in the shallow water model under the influence of surface tension and gravitational force. The authors focused on finding the multiple-soliton solutions using the simplest equation method. Moreover, the stability of the order differential equations form of the DS system has been investigated without including the effect of the angular frequency on the stability classifications.

The aim of the present work is to study the DSEs for surface waves propagating at a finite depth of water open to air to get more results in addition to those obtained earlier. The described PDEs of this physical model are transformed into the DSEs using the reductive perturbation technique. The Painlevé test (PT) using the WTC method with Kruskal's simplification is applied to the DSEs to study their integrability and the Painlevé property. In addition, the Bäcklund transformation (BT) and Hamiltonian forms of the later equations are derived. With the aid of Mathematica software, the DSEs are solved analytically using different methods such as the BT, Hamiltonian approach (HA), and generalized and improved (G'/G)-expansion methods to describe the wave motion. The dispersion analysis and conservation laws of the DSEs as well as the stability of its ordinary differential equation (ODE) by including the effect of angular frequency form are investigated. Furthermore, the effect of the Ursell parameter, which depicts the competing effects between the dispersion and nonlinearity, and the total constant of energy on the wave profiles in shallow water are illustrated.

The paper is constructed as follows. In Section 2, the water wave model is described and the case of long waves is investigated. In addition, the derivation of the DSEs which is slightly different from those introduced in the previous works [3,4,39] is presented. Conservation laws of the DSEs are derived. In Section 3, the WTC-Kruskal algorithm of the PT is briefly introduced to check the integrability of the DSEs, then the Painlevé truncation expansion is used to obtain the BT. Further, several new traveling wave solutions of the DSEs are obtained by the BT, HA and (G'/G)-expansion methods. In Section 4, the stability of the obtained ODE form of the DSEs is analyzed using the phase plane method. Section 5 discusses the results, and the conclusions are reached in Section 6.

2. The model

In the standard approach of the shallow water wave problems, the fluid is supposed to be inviscid and incompressible and its motion to be irrotational. Therefore, the wave motion is represented by the velocity potential $\varphi(x, y, z, t)$ and water surface elevation $\eta(x, y, t)$ which satisfy the Laplace equation and the boundary conditions. The system of equations for ϕ, η and their derivatives can be found in many textbooks [2–7,39].

It is convenient to study the shallow water wave problem in non-dimensional form according to the following dimensionless variables that read

$$\bar{x} = \frac{x}{\lambda}, \quad \bar{y} = \frac{y}{\lambda}, \quad \bar{z} = \frac{z}{h}, \quad \bar{t} = \frac{c}{\lambda}t, \quad \bar{\eta} = \frac{\eta}{a}, \quad \bar{\phi} = \sqrt{\frac{h}{g(\lambda a)^2}}\phi, \tag{1}$$

where $z = h$ is the undisturbed water depth (z-axis is in the vertical direction), λ is a typical wave length of the surface waves, and a is a typical amplitude of the surface wave. Therefore, the non-dimensional form (without the bars) of the set of hydrodynamic equations for the (2+1)-dimensional flow reads:

$$\delta\left(\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2}\right) + \frac{\partial^2\phi}{\partial z^2} = 0, \qquad 0 < z < 1 + \varepsilon\ \eta(x, y, t), \tag{2}$$