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Dynamics-based analytical solutions to singular integrals for elastodynamics by time domain boundary element method

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ABSTRACT

The singularities in 2-D time domain boundary element (TD-BEM) formulation for elastodynamics are divided into three categories: the wave front singularity, the space singularity and the dual singularity. A fully analytical procedure for dealing with the three singularities is proposed by adopting the concept of the finite part of an integral (Hadamard principle integral). In order to reduce the computation time, the conventional numerical procedure is adopted for the non-singular integrals in 2-D TD-BEM formulation. Therefore, the algorithm including the fully analytical procedure for dealing with singular integrals and the numerical procedure for dealing with non-singular integrals is implemented in this study. Two examples, 1-D rod and 2-D cavity representing the problems for the finite domain and the infinite domain respectively, are chosen to verify the effectiveness of the proposed algorithm. It shows that the results obtained from the proposed algorithm agree with the analytical solutions with good accuracy, indicating that the proposed algorithm is applicable for elastodynamics in both finite and infinite domains.

1. Introduction

The boundary element method (BEM) for elastodynamics can be classified into four categories according to the mechanism of the adopted fundamental solutions: the BEM formulations based on the static fundamental solution [1–3], the BEM formulations based on the Operational Quadrature Method [4–8], the BEM formulations based on frequency and Laplace domains [9–11] and the time domain (TD) boundary element method [12–17]. The BEM formulations based on the static fundamental solution frequently is widely used in several fields [18–20]. The BEM formulations relating to frequency and Laplace domains are often used to handle 2-D elastodynamics. Manolis and Beskos [15] extensively compared the Laplace transformation and the Fourier transformation methods. It was found the former is more effective than the latter. It was also found that the latter is suitable in linear elastic or viscoelastic problems, and needs the backward transformation [8,9]. By contrast against the above mentioned BEM relating to frequency and Laplace domains for elastodynamics, TD-BEM directly handles the elastodynamics without backward transformation.

However, the singular integral is the main problem by using TD-BEM to solve elastodynamic problems [13,15,21]. In a study on two dimensional elastic wave problem by TD-BEM, Niwa et al. [22] dealt with the elastic transient response of a cavern subjected to SH, SV and P waves, where the singular integrals were not completely solved, resulting in instability of the computation. In literatures [16] by Mansur and [23,24] by Mansur and Brebbia, the whole TD-BEM formulation

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was reported for the first time respectively for the scalar wave problem and the general elastoplastic problem. Carrer and Mansur [25] adopted Hadamard principle integral, which could be seen in the reference by Hadamard [26], to analytically solve the wave front singularity; while the space singularity and the dual singularity were numerically solved by using the method of rigid body displacement. The analytical treatments on time integral showed better modeling accuracy and computational cost than numerical ones. For the singular integrals in time, fully analytical solutions are available. Unfortunately, the available analytical treatments [13,27,28] on singularities in space are still more or less based on method of rigid body displacement, which is a concept of elastostatics. Among those important literatures, the method of rigid body displacement was employed in literatures [13,27] based on the judgment, that the singularity categories in the displacement and traction fundamental solution kernels in both the elastodynamic and elastostatic cases are the same, without the support of convincing relationships. Subsequently, based on a deduced relationship for the **G** and **H** matrices between elastostatics and elastodynamics under special conditions, which could justify the abovementioned judgment in literatures [13,27], the so-called jointed analytical-numerical treatment on singularity in space was proposed in literature [28]. Nevertheless, the method of rigid body displacement is an elastostatic concept, rather than an elastodynamic one, so, those singular treatments might not be applicable in nonlinear analysis both for elastoplastics and inelastic materials. How to comprehensively and analytically solve the singular integrals in both space and time from the viewpoint of elastodynamics for TD-BEM is still a problem. It is noted that some literatures [29,30] for analytical treatments on singularities in space domain for BEM formulation for elastostatic problems are reported, where time domain is not involved.

In current research, an analytical method by adopting Hadamard principle integral is proposed to solve the singularities in both time and space in double-layered integrals, from the viewpoint of elastodynamics by TD-BEM, rather than the viewpoint of elastostatics. Based on the analysis of the singularity of the elements of the diagonal sub-matrices in **G** and **H** matrices, it is found that the essential influencing factors on the singularity are two integral coefficients d_w and e_w. The integral coefficients d_w and e_w are analytically solved, by using the Hadamard principle integral to eliminate the singularity, for all the three time-space integration ranges. The proposed method for analytically eliminating the singularity for 2-D elastodynamics is verified by two examples, one for 1-D rod in a finite medium and the other one for 2-D circular cavity in an infinite medium under Heaviside-type uniform loads.

2. Boundary integral equations in terms of displacement for elastodynamics

The algorithm presented in this paper is for the plane strain problem without body force. By replacing the Young's Modulus E and the Poisson's ratio ν, respectively, with $(1+2\nu)E/(1+\nu)^2$ and $\nu/(1+\nu)$ in the expression of Lame constant λ in this paper, while keeping the Lame constant μ (the shear modulus) unchanged, the algorithm for the plane strain problem in this paper is transformed into the algorithm for the plane stress problem. For the elastodynamics with constant body force, such as gravity, the algorithm in this paper is still applicable by including an additional boundary integral.

The displacement fundamental equation for the plane strain elastodynamics can be formulated as:

$$c_{ik}u_i(P,t) = -\int_\Gamma\int_0^t p^*_{ik}(P,\tau;Q,t)u_k(Q,\tau)d\tau d\Gamma + \int_\Gamma\int_0^t u^*_{ik}(P,\tau;Q,t)p_k(Q,\tau)d\tau d\Gamma \tag{1}$$

where u^*_{ik} and p^*_{ik} respectively represent the displacement and traction fundamental solutions at the field point Q in k direction at t instant, due to the unit impulse at the source point P in i direction at any time instant τ; t and τ respectively stand for the analysis time instant and the time instant that the impulse starts the impact.

The displacement and traction fundamental solutions are expressed by Eqs. (2) and (3), respectively, as:

$$u^*_{ik}(X,t;\xi,\tau) = \frac{1}{2\pi\rho c_s}\begin{bmatrix}\left(E_{ik}L_s + F_{ik}L_s^{-1} + J_{ik}L_sN_s\right)H_s \\ -\frac{c_s}{c_d}\left(F_{ik}L_d^{-1} + J_{ik}L_dN_d\right)H_d\end{bmatrix} \tag{2}$$

$$p^*_{ik}(P,\tau;Q,t) = \frac{1}{2\pi\rho c_s}\begin{Bmatrix}A_{ik}\left(rL_s^3H_s + L_s\frac{\partial H_s}{\partial(c_s\tau)}\right) + B_{ik}L_sN_sH_s + \frac{D_{ik}}{r^2}\left(r^3L_s^3H_s + L_sN_s\frac{\partial H_s}{\partial(c_s\tau)}\right) \\ -\frac{c_s}{c_d}\left[B_{ik}L_dN_dH_d + \frac{D_{ik}}{r^2}\left(r^3L_d^3H_d + L_dN_d\frac{\partial H_d}{\partial(c_d\tau)}\right)\right]\end{Bmatrix} \tag{3}$$

The notations ρ and r in Eqs. (2) and (3) stand for density and the distance betwee*n* the source point P and the field point Q, respectively, while c_s and c_d represent the secondary and the primary wave velocities, respectively. $H_s$and H_d are Heaviside functions.

Therefore, the two integral terms on the right hand side of Eq. (1) are expressed by Eqs. (4) and (5), respectively, as:

$$\int_\Gamma\int_0^t p^*_{ik}u_kd\tau d\Gamma = \frac{1}{2\pi\rho c_s}\int_\Gamma\begin{bmatrix}(A_{ik}+D_{ik})\left|\int_0^t rL_s^3u_kH_sd\tau\right. + B_{ik}\int_0^t L_sN_su_kH_sd\tau \\ -\frac{c_s}{c_d}\left(B_{ik}\int_0^t L_dN_du_kH_dd\tau + D_{ik}\left|\int_0^t rL_d^3u_kH_dd\tau\right.\right)\end{bmatrix}d\Gamma \tag{4}$$

$$\int_\Gamma\int_0^t u^*_{ik}p_kd\tau d\Gamma = \frac{1}{2\pi\rho c_s}\int_\Gamma\int_0^t\begin{bmatrix}\left(E_{ik}L_s + F_{ik}L_s^{-1} + J_{ik}L_sN_s\right)H_s \\ -\frac{c_s}{c_d}\left(F_{ik}L_d^{-1} + J_{ik}L_dN_d\right)H_d\end{bmatrix}p_kd\tau d\Gamma \tag{5}$$