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On characterizing spatially variable soil shear strength using spatial average

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ABSTRACT

The purpose of this study is to examine in more detail under what conditions would spatial averaging over some prescribed region be sufficient to reproduce the response statistics arising from a spatially variable field. The spatially variable undrained shear strength is first simulated by a random field, and the actual response of a spatially variable clay in three problems (soil column, retaining wall, shallow foundation) is simulated using the random finite element method. This actual response is then compared to the spatial average response (the response of a homogeneous clay whose undrained shear strength is equal to certain spatial average). It is observed that the actual response can be well approximated by the spatial average response only for situations where the critical slip curve is constrained. This constraint is the most significant for the retaining wall and the least significant for the soil column.

1. Introduction

Soil-structure interaction occurs over a finite volume of soil (influence zone). For a spatially variable soil mass, it is natural to examine if it can be simplified as an equivalent homogeneous soil mass. It is possible that the equivalency, if it exists, depends on the nature of the spatial variability and the type of the response (e.g. capacity or deformation). The simplest equivalency is to convert the spatially variable soil parameter into a homogeneous spatial average [1,2]. Fenton and Griffiths [3] studied the settlement of a footing on a three-dimensional (3D) spatially variable soil mass with this practical objective in mind. Their results showed that the effective elastic modulus can be well represented by the geometric average within an influence zone under the footing. Honjo and Otake [4] studied the capacity of a footing on a two-dimensional (2D) spatially variable soil mass. Their results showed that the effective shear strength can be well represented by the spatial average within an influence zone of different size. In structural mechanics, a similar concept of homogenization has been proposed [5–7].

In an attempt to clarify the emergent behavior of critical slip surfaces in a spatially variable soil mass, Ching and Phoon [8] found that the shear strength of a laboratory test specimen can NOT be effectively represented by spatial average over any prescribed area or curve. Instead, they found that the shear strength can be well represented only by the average over the actual slip curve. Note that the critical difference here is that the actual slip curve is not a prescribed curve, but an emergent curve that depends on the random field realization. Its trajectory changes from realization to realization, because it is the solution of a boundary value problem over a spatially variable domain. The change can be significant, for e.g., the slip curve can vary in location over the entire height of the rectangular specimen studied by [8], or the change can be limited because the curve is constrained to pass through the toe of a retaining wall studied by [9]. Even for the retaining wall problem where the slip curve is constrained, [9] found that the active lateral force generally can NOT be well represented by considering the spatial average over any prescribed area or line.

Fenton and Griffiths [3] and Honjo and Otake [4] focused on the global response of a footing (settlement, capacity), whereas [8,9] focused at a more local level on the strength mobilized along an emergent critical slip curve. It is difficult to explain why these mechanical responses, which appear similar, would produce diametrically opposite conclusions. There is a practical motivation to examine the limitations of spatial averaging, because it is obviously easier to carry out reliability-based design using a random variable (spatial average) than a random field. The purpose of this study is to examine in more detail under what conditions would spatial averaging over some prescribed region be sufficient to reproduce the response statistics arising from a spatially variable field. Clearly, the studies by [3,4] have demonstrated numerically that converting the property field of spatially variable medium into a homogeneous spatial average over a prescribed region

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works in some cases.

The property field is restricted to the undrained shear strength (s_u) in this paper. The method adopted by this paper is straightforward. Two sets of finite element method (FEM) analyses will be conducted. The first set considers a spatially variable clay whose s_u is simulated by a random field. The outcome of this first set of random finite element method (RFEM) is called the "actual response". It is the reference or actual response of the spatially variable clay. The same s_u random field is then averaged over a prescribed area or line of interest to obtain the s_u spatial average. The second set of FEM then considers a homogeneous clay whose s_u is equal to the spatial average. The outcome of this second set of FEM will be referred to as the "spatial average response". This response is then compared to the actual response simulated by the RFEM. Although two types of averages (arithmetic and geometric averages) have been studied, only the results for the geometric average will be presented. The conclusions for the arithmetic average are qualitatively similar. The comparison will be made on the following two levels: Level I compares the statistics of the two sets of responses, whereas Level II compares the two sets of responses on the 1:1 line. More specifically, the responses produced by two distinct FEM analyses are two distinct random variables. Level I checks if these random variables are "equal" in the probability distribution sense. Level II checks if these random variables are "equal" in their numerical values. Another way of viewing the distinction between Level I and Level II is that a bivariate distribution is needed to define two random variables ("actual response" and "spatial average response"). Level I only compares the similarity in the marginal distribution. Level II compares both the distribution *and the correlation* between these variables. To replace one random variable by another random variable for reliability analysis, there must be strong correlation as well. Two identically distributed independent variables cannot be used interchangeably for reliability analysis. Three problems are adopted to examine actual responses under different boundary conditions: (a) a soil column; (b) a retaining wall; and (c) a shallow foundation.

2. Random field and its simulation

In this study, the only random soil property is the soil shear strength (τ_f). The shear strength $\tau_f(x,z)$ at a point in the soil mass is simulated by a random field, where x and z are respectively the horizontal and vertical coordinates. The friction angle is taken to be 0° for simplicity, i.e., $\tau_f(x,z)=s_u(x,z)$, where s_u is the undrained shear strength. The random shear strength $\tau_f(x,z)$ is simulated as a stationary lognormal random field with inherent mean$=\mu$ and inherent coefficient of variation=COV. To define the correlation structure in $\tau_f(x,z)$ between two locations with horizontal distance$=\Delta x$ and vertical distance$=\Delta z$, the single exponential autocorrelation model is considered [1,2]:

$$\rho(\Delta x,\ \Delta z)\ =\ \exp\left(-2|\Delta x|/\delta_x-2|\Delta z|/\delta_z\right) \tag{1}$$

where δ_x and δ_z are the horizontal and vertical scales of fluctuation (SOFs), respectively. The Fourier series method (FSM) [10,11] is adopted to simulate stationary normal random fields (point process). A 2D stationary lognormal random field $\tau_f(x,z)$ over a domain of size $L_x\times L_z$ can be simulated by taking the exponential of a 2D stationary normal random field with mean$=\lambda=\ln[\mu/(1+\text{COV}^2)^{0.5}]$ and standard deviation$=\xi=[\ln(1+\text{COV}^2)]^{0.5}$:

$$\tau_f(x,\ z)=\exp\left(\lambda+\text{Re}\left[\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}(a_{mn}+ib_{mn})\exp\left(\frac{i2m\pi x}{L_x}+\frac{i2n\pi z}{L_z}\right)\right]\right) \tag{2}$$

where Re[.] denotes the real part of the enclosed complex number; a_{mn} and b_{mn} are independent zero-mean normal random variables with variance $\sigma_{mn}{}^2$ given by [10]

$$\sigma_{mn}^2=\frac{\xi^2}{q_xq_z}\left[\frac{1-\exp(-q_x)(-1)^m}{1+m^2\pi^2/q_x^2}\right]\times\left[\frac{1-\exp(-q_z)(-1)^n}{1+n^2\pi^2/q_z^2}\right] \tag{3}$$

where $q_x=L_x/\delta_x$ and $q_z=L_z/\delta_z$. Note that the auto-correlation in Eq. (1) is applied to the $\ln[\tau_f(x,z)]$ field (a normal field). Besides simulating the point process of a normal random field, the FSM is also capable of directly simulating the spatial average of the normal random field over a prescribed rectangular region (cell) in 2D [10]. To simulate a "cell average" over each element, the cell average for the underlying normal random field $\ln[\tau_f(x,z)]$ is first simulated and then the exponential of this average is used. By doing this, it is evident that the cell average for $\tau_f(x,z)$ is actually a geometric average, not an arithmetic average. In the case that the sizes of the cell are smaller than the SOFs, a geometric average is roughly the same as an arithmetic average. All averages refer to geometric averages from hereon, unless specifically specified.

3. Random finite element models

This study compares the actual response with the spatial average response for three physical problems: (a) a soil column; (b) a retaining wall; and (c) a shallow foundation. The RFEM models for these three problems are described below.

3.1. Soil column

The RFEM model for the soil column is a rectangular area of size $L_x\times L_z$=48 m × 12.8 m (Fig. 1a). The bottom boundary is supported on rollers, and the lower-leftmost node is a hinge, to prevent rigid body translation in the x direction. The unit weight of the soil is set to 0 to ensure uniform vertical stress. Young's modulus E is deterministic and equal to 400 MN/m^2, the Poisson ratio is 0.3, and the friction angle $\phi=0°$. The undrained shear strength $\tau_f(x,z)$ is simulated as a stationary lognormal random field using the FSM. The mean value μ=50 kN/m^2 and COV=0.3. The τ_f for each element is taken to be the geometric average of the $\tau_f(x,z)$ field over that element. This is equivalent to adopting the geometric average over each element. In this RFEM, two types of stress states are considered: (1) scenario C – compression test; and (2) scenario S – shear test. An increasing axial compression (scenario C) or shear stress (scenario S) is applied until the RFEM fails to converge. For scenario C, the axial stress versus strain curve (Fig. 2a) is plotted, and the yield axial stress applied on the top boundary is identified. The yield axial stress (σ_y) is identified by shifting the initial linear portion to the right (with a strain offset of 0.0001) and reading the intersect between this shifted line and the stress–strain curve (Fig. 2a). This value of σ_y, denoted by $\sigma_f{}^m$, is interpreted as the actual response for the RFEM. For scenario S, the shear stress versus strain curve (Fig. 2b) is plotted, and the yield shear stress applied on the four boundaries (τ_y) is identified. The same strain offset criterion is adopted to identify τ_y (Fig. 2b). This τ_y, denoted by $\tau_f{}^m$, is the actual response for the RFEM. This RFEM has been analyzed in [8]. This strain offset is very small, indicating a very large rigidity index, because the modulus is deliberately set to a large value (400 MN/m^2) for numerical efficiency.

For the spatial average response, the spatial averaging over the entire rectangular area of size $L_x\times L_z$, shown in Fig. 1b, is considered. The geometric average of the τ_f values for all elements is first simulated, and a homogeneous FEM is simulated to obtain the spatial average response, denoted by $\sigma_f{}^{RA}$ for scenario C and by $\tau_f{}^{RA}$ for scenario S (RA means 'rectangular average').