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Analysis of arbitrarily shaped planar cracks in two-dimensional hexagonal quasicrystals with thermal effects. Part I: Theoretical solutions

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ABSTRACT

An extended displacement discontinuity (EDD) boundary integral equation method is proposed for analysis of arbitrarily shaped planar cracks in two-dimensional (2D) hexagonal quasicrystals (QCs) with thermal effects. The EDDs include the phonon and phason displacement discontinuities and the temperature discontinuity on the crack surface. Green's functions for unit point EDDs in an infinite three-dimensional medium of 2D hexagonal QC are derived using the Hankel transform method. Based on the Green's functions and the superposition theorem, the EDD boundary integral equations for an arbitrarily shaped planar crack in an infinite 2D hexagonal QC body are established. Using the EDD boundary integral equation method, the asymptotic behavior along the crack front is studied and the classical singular index of 1/2 is obtained at the crack edge. The extended stress intensity factors are expressed in terms of the EDDs across crack surfaces. Finally, the energy release rate is obtained using the definitions of the stress intensity factors.

1. Introduction

In 1982, Shechtman observed a metallic solid (a rapidly spin-cooled alloy of 86% Al and 14% Mn) with both long-range orientational order and icosahedral point group symmetry, which was reported in 1984 [1]. This finding changed the definition of crystals [2]. Subsequently, Levine and Steinhardt [3] introduced the concept of quasicrystals (QCs) to describe this new class of ordered atomic structures with quasiperiodic, rather than periodic, translational order. Additional QCs, such as the decagonal [4], dodecagonal [5], and octagonal [6] phases, were discovered experimentally. Icosahedrite (63% Al, 24% Cu, and 13% Fe), was the first natural QC, discovered by Bindi et al. [7] in a rock sample.

Because of the special arrangement of atoms, QCs possess desirable and unique properties, such as low electrical and thermal conductivity, unusual optical properties, low surface energy and coefficient of friction, oxidation resistance, biocompatibility, and high hardness [8] in comparison with traditional materials. Due to these properties, QCs become a new class of functional and structural materials, and present many current and potential applications. For example, QCs

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can be used as a particulate-reinforcing phase for polymer- or metal-matrix composites or to form thin films/coatings for surface modifications [9,10]. On the other hand, possible application of QC materials have also been pointed out in various domains of energy savings, namely thermal insulation, light absorption, power generation and hydrogen storage [11].

For application purposes, a number of studies have been performed to determine these properties, with an emphasis on the mechanical and physical response. Based on the Landau theory [12], the elastic behavior of QCs was studied [13–16]. A generalized theory based on the laws of motion in continuum physics applicable to the elasticity of all QCs was established by Ding et al. [17]. It is noted that because of its quasi-periodic symmetry, concepts of multi-field or high-order theory were introduced instead of the classical crystallographic theory. The phonon field represents the lattice vibrations while the phason field depicts the quasi-periodic rearrangement of atoms, and both fields are used simultaneously to describe the elasticity of QCs. Shi obtained conservation laws of a three-dimensional (3D) solid periodically stacked in a two-dimensional (2D) quasi-periodic structure with decagonal symmetry using a special version of Noether's theorem [18]. Fan reviewed the development of the study in mathematical theory and methods of mechanics of QCs, including elasticity, plasticity, defects, dynamics, and fracture [19].

Defects such as cracks, cavities, and inclusions can greatly affect the optical, magnetic, thermal, and mechanical properties of QCs. Wang and Pan studied defects in one-dimensional (1D) hexagonal and 2D octagonal QCs [20]. It should be noted that 2D QCs are defined as 3D bodies in which the atom arrangement is quasi-periodic in a plane and periodic in the orthogonal direction. Li and Fan studied the plasticity and developed a generalized Dugdale model of 2D decagonal QCs [21]. Gao and Ricoeur studied the effect of a spheroidal quasicrystalline inclusion embedded in an infinite matrix consisting of 2D hexagonal QCs subjected to uniform loadings [22]. The results presented by Gao and Ricoeur can be applied to the limiting cases involving in homogeneities including rigid inclusions, cavities, and penny-shaped cracks. By introducing four displacement functions, Yang et al. obtained the general solution of 3D thermoelasticity for 2D hexagonal QCs and investigated a uniformly distributed temperature change on the crack face of a penny-shaped crack [23,24]. Using the generalized potential theory method, Wang et al. investigated the mode I crack problem in 2D hexagonal QCs [25]. Using similar techniques, Li et al. studied the problem of a penny-shaped crack in a half-infinite plane [26]. However, the studies described above were limited to simple cases of penny-shaped cracks or uniform loadings.

The displacement discontinuity method (DDM) is an efficient way to analyze and study crack problems. This method captures the intrinsic characteristic of a crack across which the physical fields are discontinuous. The DDM was extended to solve crack problems in elastic [27,28], piezoelectric [29,30], piezoelectric semiconductors [31], and magneto-electro-elastic media [32,33]. For QCs, Fan et al. used the extended displacement discontinuity method (EDDM) to analyze the fracture of 1D hexagonal QCs with piezoelectric effects [34].

In this work, the EDDM is extended to analyze 3D thermo-phonon-phason planar crack problems in 2D hexagonal QCs. The paper is organized as follows. In Section 2, fundamental equations, according to the 2D QC linear elastic theory established by Ding et al., are given for static thermoelastic fields of 2D hexagonal QCs. General solutions satisfying these basic equations are presented in Section 3. In Section 4, Green's functions of unit point EDDs are derived. In Section 5, the EDD boundary integral equations are established. In Section 6, the extended stress singularities for an arbitrary planar crack in a 2D hexagonal QC medium are analyzed. The energy release rate is derived in Section 7. Finally, conclusions are drawn in Section 8.

2. Fundamental equations for 2D hexagonal QCs with thermal effects

Consider 2D hexagonal QCs with atoms arranged periodically in the z direction and quasi-periodically in the horizontal planes parallel to the plane oxy in the Cartesian coordinate system $oxyz$. In the absence of a body force and body heat source, the governing equations of the 2D hexagonal QC with thermal effects are given by [17,23]

$$\begin{aligned}
&\frac{\partial\sigma_{xx}}{\partial x}+\frac{\partial\sigma_{xy}}{\partial y}+\frac{\partial\sigma_{zx}}{\partial z}=0,\\
&\frac{\partial\sigma_{xy}}{\partial x}+\frac{\partial\sigma_{yy}}{\partial y}+\frac{\partial\sigma_{yz}}{\partial z}=0,\\
&\frac{\partial\sigma_{zx}}{\partial x}+\frac{\partial\sigma_{yz}}{\partial y}+\frac{\partial\sigma_{zz}}{\partial z}=0,
\end{aligned}\tag{1a}$$

$$\begin{aligned}
&\frac{\partial H_{xx}}{\partial x}+\frac{\partial H_{xy}}{\partial y}+\frac{\partial H_{xz}}{\partial z}=0,\\
&\frac{\partial H_{yx}}{\partial x}+\frac{\partial H_{yy}}{\partial y}+\frac{\partial H_{yz}}{\partial z}=0,
\end{aligned}\tag{1b}$$

$$\frac{\partial h_{x}}{\partial x}+\frac{\partial h_{y}}{\partial y}+\frac{\partial h_{z}}{\partial z}=0,\tag{1c}$$

where σ_{ij}(H_{ij}) and h_i, with $i, j = x, y, z$, are the components of phonon (phason) stress and heat flux, respectively. These are referred to as the extended stresses in the present paper. The linear constitutive equations of 2D hexagonal QCs take the