Applied Mathematical Modelling 55 (2018) 205–223

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Dynamics regularization with tree-like structures

Rochelle Nieuwenhuis[a], Madoka Kubota[b], M.R. Flynn[c,*], Masayuki Kimura[d], Takashi Hikihara[d], Vakhtang Putkaradze[a]

[a] Department of Mathematical and Statistical Sciences, University of Alberta Edmonton, AB T6G 2G1, Canada
[b] Fuji Electric, Power Electronics Technology Development Center, Product Technology Laboratory, 1, Fuji-machi, Hino-city, Tokyo 191-8502, Japan
[c] Department of Mechanical Engineering, University of Alberta, Edmonton, AB T6G 2G8, Canada
[d] Department of Electrical Engineering, Kyoto University, Katsura, Kyoto 615-8510, Japan

ARTICLE INFO

Article history:
Received 9 March 2017
Revised 16 October 2017
Accepted 26 October 2017
Available online 11 November 2017

Keywords:
Nonlinear dynamics
Coupled nonlinear oscillators
Tree-like structures
Dynamics regularization

ABSTRACT

The dynamics of a fully nonlinear, tree-structured resonator and its response to a broadband forcing of the branches is examined. It is shown that the broadband forcing yields a transfer of energy between the parts of the spectrum so that the spectrum becomes progressively more narrow-band for each level of the tree-like structure in the direction of the stem. We show that this behavior is in contrast to the response of a linear oscillator, which simply filters out the harmonics away from the resonance. We term such behavior "regularization" and examine its significance for two- and three-dimensional motion using a Lagrangian framework. Key to our analysis is to investigate the dependence of the spectrum of motion, and its narrowing, on the parameters of the tree-like structure, for instance the lengths of different branches. Model predictions are obtained for idealized wind forcing characterized by an airflow that is interrupted at random time intervals. Our numerically-derived results are then compared against the data collected from select analogue laboratory experiments, which confirm the robust nature of the vibration regularization.

1. Introduction and literature review

The dynamics of connected non-linear resonators has played an important role in both fundamental science and engineering [1]. On the application side, the question of the response of such systems to strongly non-harmonic or non-constant forcing has been one of the main inspirations driving the development of the field. The case of energy absorption from incoherent, random forces is known more generally as *broadband harvesting* [2]. Mechanical devices constructed for the purpose of utilizing broadband sources are normally based on a set of moveable objects positioned on a flexible piezoelectric element [3]; they convert the motion of the underlying substrate (*e.g.* vibration) into the deformation of a piezo element and thereby produce energy. The efficiency and power output of these devices depends to a large extent on the nature of motion of the power-generating element, as well as other factors. Piezo-electric devices achieve the highest efficiency when they operate within an optimal frequency band, and the efficiency drops rapidly outside this band [4–8]. To focus the ambient forcing into the motion of the resonator in the optimal frequency band, strong nonlinearity, bi-stability and chaotic

* Corresponding author.
E-mail address: mrflynn@ualberta.ca (M.R. Flynn).

https://doi.org/10.1016/j.apm.2017.10.039
0307-904X/

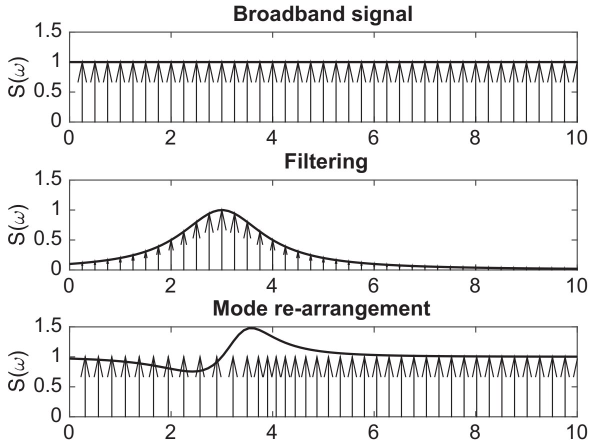

Fig. 1. A schematic illustration of linear filtering vs. regularization. A filter applied to an initially broadband signal with a flat spectrum will multiply the amplitude of each harmonic with the frequency response function. On the other hand, regularization re-shuffles the position on the harmonics making them more dense in some areas, thereby increasing the spectral density amplitude in these same areas.

responses remain topics of active investigation [9–18]. For larger devices, electromagnetic induction has been used to harvest narrow-band random excitation by exploiting highly nonlinear resonances [19,20]. Recent work related to *vibro-wind* energy generation explores the combined dynamics of a chain of such harvesters due to fluid–structure interactions [21], the oscillations induced by vortex shedding behind arrays of flexible structures [22] and increased energy absorption from coupled Duffing oscillators [23].

In this paper, we show that it is possible to narrow the spectrum of broadband forcing using the nonlinear dynamics associated with coupled resonators arranged in a tree-like configurations. We focus on the dynamics of tree-like structures as a response to broadband forcing leaving the precise application of these ideas to energy harvesting to future works. We demonstrate that under quite general conditions, a broadband forcing applied to the top branches regularizes towards the stem, *i.e.*, the stem shows substantially more regular motion with a narrow bandwidth of the spectrum. In terms of spectral analysis of the signal for each level of the tree-like structure, such an effect would move energy between harmonics, mostly from the higher harmonics to the lower ones, making the spectrum more concentrated around a certain frequency. We show that this phenomenon comes from nonlinear effects, and is in contrast to a linear resonator, which acts as a filter centered around a certain frequency. Intuitively, this can be formulated as follows. Suppose, for simplicity, that we consider a completely white spectrum absorbed by the device. This is illustrated in Fig. 1 by the flat line and a sample of harmonics with given amplitude, shown as arrows. For each frequency, a linear filter like a resonating linear oscillator will simply multiply the amplitude of the spectrum of the harmonics by a certain function. On the other hand, re-arrangement of the modes, which is a purely nonlinear effect, will reposition the location of the harmonics, centering the sampled arrows around a given frequency (or perhaps several frequencies), thus making the spectral density higher at this point. These are fundamentally different concepts, as can be seen from the Fig. 1. In reality, both effects will be present in any realistic system. Regularization having nonlinear effects as a leading cause of band-narrowing will preserve much more of the high harmonic content than a typical linear filter.

Formally, we define band-narrowing as the transformation of a signal when more power in the spectrum is moved to lower bandwidths and concentrated about a single frequency. While this definition is somewhat inexact, we note that a precise mathematical definition of this notion, applicable to a broad range of physical cases, appears elusive. An important consequence of our approach is that the band-narrowing, or regularization, is achieved due to the structure of the system itself, without requiring external control mechanisms.

Although the associated quantitative details will be shown to be nontrivial, the conceptual foundation is simple. Consider, for instance, a tree swaying in the wind. Whereas the movement of the trunk is typically quite regular, the motion of the branches is much less so. Unfortunately, energy extraction from real trees is difficult: recent investigations [24,25] of life-sized trees yielded quite modest power generation, *i.e.* 44 mW from the trunk and several Watts for the motion of entire (several meter tall) tree including the top branches. This rather disappointing result follows from the comparatively small deviations of the tree trunk from its equilibrium position, an evolutionary adaption presumably meant to minimize the likelihood of trunk fracture during violent wind storms. We therefore aim to construct artificial tree-like structures with characteristics that are in some sense opposite to real trees, providing potentially large amplitude displacements and an efficient regularization of the forcing.

In this paper, we use the term “trees” in the generalized sense, both for structures possessing a branched structure analogous to real trees and, alternatively, for coupled sequences of oscillators that exhibit a tree-like dynamical diagram. The former type is more useful for large-scale, low-frequency harvesting *e.g.* by wind or waves whereas the latter is more relevant for miniaturized harvesting devices. Of course, a multi-level structure exhibiting branching has many resonances