Probabilistic Engineering Mechanics 45 (2016) 115–126

Contents lists available at ScienceDirect

Probabilistic Engineering Mechanics

journal homepage: www.elsevier.com/locate/probengmech

Random function based spectral representation of stationary and non-stationary stochastic processes

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A R T I C L E I N F O

Article history:
Received 7 October 2015
Received in revised form
1 April 2016
Accepted 14 April 2016
Available online 16 April 2016

Keywords:
Spectral representation
Random function
Random constraint
Seismic acceleration
Probability density evolution
Structural reliability

A B S T R A C T

In conjunction with the formulation of random functions, a family of renewed spectral representation schemes is proposed. The selected random function serves as a random constraint correlating the random variables included in the spectral representation schemes. The objective stochastic process can thus be completely represented by a dimension-reduced spectral model with just few elementary random variables, through defining the high-dimensional random variables of conventional spectral representation schemes (usually hundreds of random variables) into the low-dimensional orthogonal random functions. To highlight the advantages of this scheme, orthogonal trigonometric functions with one and two random variables are constructed. Representative-point set of the dimension-reduced spectral model is derived by employing the probability-space partition techniques. The complete set with assigned probabilities of points gains a low-number-sample stochastic process. For illustrative purposes, the stochastic modeling of seismic acceleration processes is proceeded, of which the stationary and non-stationary cases are investigated. It is shown that the spectral acceleration of simulated processes matches well with the target spectrum. Stochastic seismic response analysis, moreover, and reliability assessment of a framed structure with Bouc-Wen behaviors are carried out using the probability density evolution method. Numerical results reveal the applicability and efficiency of the proposed simulation technique.

1. Introduction

It is well understood in engineering community that the rational description and modeling of random excitations underlies the analysis and reliability assessment of engineering structures. The systematic development on this study began with the contribution of Housner in 1947 [11], who modelled the seismic acceleration as a pulse-structured stochastic process. While the accurate stochastic analysis of structures, in practices, involves the logical modeling of engineering excitations. The practical demand highly prompts the enthusiasm of researchers on the simulation of stochastic processes. There has arisen tens of simulation techniques so far among which, nevertheless, the scheme of spectral representation is widely used due to its rigorous mathematical formulation and easier-to-be-implemented algorithm.

The concept of spectral representation is original from the pioneered work of Rice [29], Goto and Toki [14], Borgman [1] upon the simulation of a one-dimensional stochastic process using harmonic superposition method. It is Shinozuka who completely proposed the general principles of stochastic process simulation employing spectral representation schemes [31,34]. In the following 40 years, a lot of researchers were devoted into this field. With these consistent efforts, the spectral representation scheme has finally been accepted by engineering communities and been used in practical applications. As regards the critical advances of the spectral representation scheme, Shinozuka and Deodatis first derived the theoretical formulation for the one-dimensional stochastic processes with single variable [32], and later extended the theoretical formulation to the high-dimensional Gaussian random fields [33]. Deodatis then investigated the multi-variant stationary process featuring the ergodic behaviors [8]. Simultaneously, he suggested the spectral representation-based simulation algorithm to generate sample functions of a non-stationary, multi-variate stochastic process with evolutionary power spectra [9]. Spanos and Zeldin addressed the characteristics of sample functions of spectral

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http://dx.doi.org/10.1016/j.probengmech.2016.04.004
0266-8920/

representation scheme, of which the computational efficiency and applicability were included as well [35]. Liang et al. developed a spectral representation scheme upon the non-stationary seismic ground motions directly using the representation theory of time-varying spectrum of non-stationary stochastic process, where the sample processes were yielded through constructing a cosine-function series [23]. Cacciola and Deodatis proposed a spectral-representation-based methodology for deriving fully non-stationary and spectrum-compatible ground motion vector processes [2].

The Karhunen–Loeve expansion is also used to represent both stationary and non-stationary stochastic processes. Ghanem and Spanos dealt with the Karhunen–Loeve expansion in the context of stochastic finite elements [13]. Huang et al. analyzed the Karhunen–Loeve expansion as a simulation tool for both stationary and non-stationary Gaussian processes focusing on convergence and accuracy [16]. Phoon et al. simulated non-Gaussian processes with a given marginal distribution and with a given covariance function [25]. They later improved the non-Gaussian simulation technique by prescribing a fractile covariance function [26]. Grigoriu evaluated the Karhunen–Loève expansion and the spectral representation coinciding for weakly stationary processes [15]. It is noted that the Karhunen–Loeve expansion requires the solution of an eigenvalue problem, which has not the advantage over the spectral representation scheme merely with an FFT for simulating stationary processes. While the Karhunen–Loeve expansion does not require the existence of a spectral density function, i.e. it is ready-made for non-stationary processes.

Although the spectral representation scheme provides a logical theorem and a reliable result, which is limited however by its computational cost in case of the random modeling of engineering excitations. In practices, hundreds of random variables associated with phase angles are usually required for securing an accepted accuracy. This logical structure limits the applicability of spectral representation scheme. Recently, Chen et al. developed an updated spectral presentation scheme through constructing so-called random harmonic functions [5]. In their work, the summation of a few number of components of random harmonic functions can derive the consistent spectrum to the objective spectral function. This scheme was subsequently enhanced with the optimization of frequency points in spectral function [4]. While the statistical compatibility of sample processes derived from the updated scheme still remain open.

This paper aims to developing a family of spectral representation schemes with just one or two elementary random variables through defining the high-dimensional orthogonal random variables of classical spectral representation into the low-dimensional orthogonal random functions. The highlight is that using the probability-space partition techniques of random variables, a complete set of sample processes with assigned probabilities can be deduced from the power spectral density or its time-varying counterpart. This treatment significantly reduces the number of sample functions. It prompts, meanwhile, a ready integration with the probability density evolution method (PDEM) resulting in an efficient scheme upon the stochastic responses analysis and reliability assessment of nonlinear structures. The remaining sections arranged in this paper are distributed as follows. Section 2 revisits the spectral representation theorem of stationary and non-stationary stochastic processes, where two families of spectral representation schemes are addressed. The transform of high-dimensional orthogonal random variables of classical spectral representation into the low-dimensional orthogonal random functions is detailed in Section 3. Stationary and non-stationary seismic processes, for illustrative purposes, modeled by the renewed spectral representation scheme are addressed in Section 4. Cases study on stochastic response analysis and reliability assessment of nonlinear structures subjected to random seismic processes are included in Section 5, respectively. Probability density evolution method is employed in the case study. The concluding remarks are included in Section 6.

2. Conventional spectral representation of stochastic process

As indicated in the previous work, the sample functions of one-dimensional and univariate non-stationary stochastic processes can be derived by integrating the Priestley's evolutionary spectral representation theory [23,27,28]. A one-dimensional, real-valued, univariate non-stationary process $X(t)$ with zero mean and two-sided evolutionary power spectral density function $S_X(\omega, t) = |A(\omega, t)|^2 S(\omega)$, where $A(\omega, t)$ denotes a deterministic modulating function of both t and ω, could thus be expressed as the following integral formulation:

$$X(t) = \int_0^{\infty} \cos(\omega t)\mathrm{d}U_t(\omega) + \sin(\omega t)\mathrm{d}V_t(\omega) \tag{1}$$

where $U_t(\omega)$ and $V_t(\omega)$ denote the spectral-process components of the real-valued non-stationary process $X(t)$. Their increments $\mathrm{d}U_t(\omega)$ and $\mathrm{d}V_t(\omega)$ must satisfy the following conditions:

$$E\left[\mathrm{d}U_t(\omega)\right] = E\left[\mathrm{d}V_t(\omega)\right] = 0,\ \omega \ge 0 \tag{2}$$

$$E\left[\mathrm{d}U_t(\omega)\mathrm{d}U_t(\omega)\right] = E\left[\mathrm{d}V_t(\omega)\mathrm{d}V_t(\omega)\right] = 2S_X(\omega, t)\mathrm{d}\omega,\ \omega \ge 0 \tag{3}$$

$$E\left[\mathrm{d}U_t(\omega)\mathrm{d}U_t(\omega')\right] = E\left[\mathrm{d}V_t(\omega)\mathrm{d}V_t(\omega')\right] = 0,\ \omega,\ \omega' \ge 0,\ \omega \ne \omega' \tag{4}$$

$$E\left[\mathrm{d}U_t(\omega)\mathrm{d}V_t(\omega')\right] = 0,\ \omega,\ \omega' \ge 0 \tag{5}$$

where $E[\cdot]$ indicates the mathematical expectation; $S_X(\omega, t)$ denotes the double-sided evolutionary power spectral density function of real-valued non-stationary process $X(t)$.

Eq. (1) could be written in the discrete form as follows:

$$X(t) \approx \sum_{k=0}^{\infty} \left[\cos(\omega_k t)\Delta U_t(\omega_k) + \sin(\omega_k t)\Delta V_t(\omega_k)\right] \tag{6}$$

where $\omega_k = k\Delta\omega$, and $\Delta\omega$ should be small sufficiently but finite so that the discrete form Eq. (6) mathematically approximates to the integral form Eq. (1). Obviously, the increments $\Delta U_t(\omega_k)$and $\Delta V_t(\omega_k)$should satisfy the basic condition; say (Eqs. (2)–(5)).

If the increments $\Delta U_t(\omega_k)$ and $\Delta V_t(\omega_k)$are defined as

$$\Delta U_t(\omega_k) = A_{k,t}X_k,\ \Delta V_t(\omega_k) = A_{k,t}Y_k \tag{7}$$

$$A_{k,t} = \sqrt{2S_X(\omega_k, t)\Delta\omega},\ \omega_k = k\Delta\omega \tag{8}$$

where $\{X_k, Y_k\}$ denotes a set of standard orthogonal random variables, and submits to the rules as follows:

$$E\left[X_k\right] = E\left[Y_k\right] = 0,\ E\left[X_j Y_k\right] = 0,\ E\left[X_j X_k\right] = E\left[Y_j Y_k\right] = \delta_{jk} \tag{9}$$

where δ_{jk} denotes the Kronecker–Delta function.

Substituting (Eqs. (7) and (8)) into Eq. (6), and taking a finite series representation, the non-stationary process $X(t)$ can be represented by the following finite series:

$$\hat{X}_{N,1}(t) = \sum_{k=0}^{N-1} \sqrt{2S_X(\omega_k, t)\Delta\omega}\left[\cos(\omega_k t)X_k + \sin(\omega_k t)Y_k\right] \tag{10}$$

where $\hat{X}_{N,1}(t)$ denotes the simulated non-stationary process. It is seen from (Eqs. (6) and (10)) that the original stochastic process