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Modelling post-crack tension-softening behavior of fiber-reinforced materials

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ABSTRACT

We present a general method for the traction-separation law for the cohesive model of fiber reinforced materials with brittle matrix. The proposed approach is based on results from the theories of marked point and fiber processes. The application of stochastic notions in the field of traction-separation laws and tension-softening curves for fiber reinforced composites allows the thorough investigation of the random effect of the fiber reinforcement on cohesive behavior. The presented method accounts for correlations between length and orientation as may be the case in real fiber reinforced composites. We study the influence of randomness of fiber length and degree of anisotropy on the post-crack tension softening curves. It turns out that fiber length and orientation distributions have a tremendous effect on the crack-opening behavior.

1. Introduction

Traction-separation laws are used for numerical and analytic studies of the crack propagation for different load cases and various length scales. They describe the failure behavior of materials and are often deployed in finite element analyses with predefined crack paths. These traction-separation laws include the cohesive stress at the crack plane σ_c and the crack width opening d. The resulting coherence $\sigma_c = \sigma_c(d)$ is called in the following *tension-softening curve* (TSC). The measurement of such a TSC is a formidable task and was carried out in the past for various materials [1]. Due to the stochastic character of composite materials with fiber reinforcement [2] traction-separation laws represent a very helpful tool to investigate failure behavior of such materials.

The classical paper [3] presented a model which allows the computation of force resistance of reinforcing fibers bridging cracks in brittle matrix fiber reinforced composites (FRC) under the assumption that the fibers have constant length and are isotropically orientated. This work has been continued in many subsequent scientific investigations (e.g. [4–7]).

However, in real FRC the fiber lengths are never constant. Fiber length is affected by various factors such as processing of fibers (cutting, chemical and mechanical treatments), embedding and processing of reinforced material (mixing, casting, etc.). Refs. [8] and [9] proposed several statistical distributions for random fiber lengths.

In real structures also deviations from the isotropic orientation distribution of fiber directions appear. While it seems to be natural to expect some kind of anisotropy of fiber orientation in the case of long-fiber reinforced materials, recent studies showed that in some composite materials even short fibers are not always isotropically oriented [10–12].

Finally, due to casting process, buoyancy effects and sedimentation, it has to be assumed that in various composites lengths and orientations of fibers are correlated. However, the joint influence of randomness of length and orientation of fibers in composite materials on mechanical properties has never been studied so far, although the spatial distribution of fibers in composite materials can be determined statistically (see e.g. [13]).

In view of the general aim of improving or optimizing material properties, the influence of randomness of fiber length and orientation on mechanical properties is of great interest. It is natural to ask: How will the post-crack TSC vary if the fiber length is not constant but the mean fiber length is fixed? To which extent does the TSC change if the fibers have some special direction distribution and are not isotropically oriented?

Obviously, there is a simple qualitative answer: Random fiber lengths imply that there are fibers which are longer than the mean fiber length. Therefore it might be expected that random fiber lengths lead to a TSC higher than for constant fibers. Furthermore,

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due to friction of fibers during pull-out, TSC will indeed change if the orientation of the fibers is not isotropic. Both effects are studied in the present paper.

In order to demonstrate the application of our theory we use results from the literature about the frictional bond between fiber and matrix such as [14–16]. With the aid of such measurements the post-crack TSC can be studied quantitatively.

The paper is organized as follows. The methods and assumptions made in order to derive the traction-separation law are presented in Section 2. There we define the mathematical notions, present an equation for TSC for general (joint) distributions of fiber length and orientation and also point out further extensions of the model. Finally, we investigate the impact of correlation of fiber length and orientation on the post-crack TSC in Section 4.

2. Model assumptions and methods

2.1. Model assumptions and extensions

Throughout the paper we make the following model assumptions, which are mainly standard in the relevant literature, but some extensions are new.

We consider a statistically homogeneous [17, p. 28] matrix material with randomly distributed fibers under the following conditions, see e.g. [3]: The matrix is discontinued by a planar crack of width d, see Fig. 1, its deformation during the fiber pull-out is neglected. The spatial distribution of the positions of fibers in the composite is homogeneous and is independent of fiber length and orientation, the fibers are straight with cylindrical geometry. They behave linear elastically and rupture if their axial stress reaches the fiber strength $\sigma_{f,\max}$. The Poisson effect of the fibers during pull-out is neglected, the fiber–matrix bond is frictional and the elastic bond strength is neglected.

As the crack opening d increases, the fiber ends are pulled out of the matrix. Eventually one fiber end is pulled out completely or the fiber ruptures due to high tension.

Additionally we assume that fiber length and orientation are random. We describe this randomness by a two-dimensional probability density function (p.d.f.). This means that fiber length and fiber orientation are allowed to depend on each other, which allows a realistic approximation of a wide class of composite materials where one phase is built of fibers. Our approach can easily be combined with various models concerning the fiber pull-out mechanism.

2.2. Theoretical background, distribution of intersecting fibers

In order to fix notation we describe fiber systems and their characteristics in what follows.

A system of fibers is a spatial set of line segments. Each fiber is described by a reference point, length and angle w.r.t. the normal of some given crack plane. For convenience the crack plane is assumed to be the (x,y)-plane and the reference point of each fiber is its top point (in the sense of the z-axis). The fiber angle is the polar angle β of the fiber. Fig. 1 shows the underlying geometry of a single fiber that intersects a crack plane A_c with inclining angle β and embedded residual lengths r_1 and r_2.

The random set of fibers is described by the following summary characteristics:

1. N_{sp} – mean number of fibers (i.e. fiber reference points) per unit volume.
2. $f_{sp,L,B}(l,\beta)$ – joint p.d.f. of fiber length l and polar angle β in space.

These characteristics are often known a priori and they are measurable e.g. by computed tomography, see [18,19], and [13,20].

If angles β and lengths l are statistically independent the joint p.d.f. is the product of the univariate p.d.f. $f_{sp,L}(l)$ and $f_{sp,B}(\beta)$ of length and angle.

The parameter N_{sp} belongs to a group of summary *mean-value* characteristics, which include also fiber volume fraction V_f, mean fiber length $\bar{l}$ and fiber cross-sectional area A_f. They satisfy the equation

$$N_{sp} = \frac{V_f}{\bar{l}A_f}. \tag{1}$$

After the formation of a planar crack, i.e. when $d=0$, we are interested in the random embedded residual fibre lengths and the inclination angle w.r.t. the normal of the crack plane of the fibers which intersect the crack, see Fig. 1. We describe these quantities by the characteristics

1. N_{pl} – mean number of fibers intersecting the crack plane per unit area and
2. $f_{pl,R_1,R_2,B}(r_1,r_2,\beta)$ – joint p.d.f. of residual lengths above and below the crack plane and polar inclination angle.

There are close mathematical relationships between these plane-related characteristics and the space-related characteristics of the fiber system. In particular, we have (cf. [21, Section 8.4])

$$N_{pl} = N_{sp}\mathbb{E}(L_{sp}\ \cos\ B_{sp}). \tag{2}$$

In this equation the expression

$$\mathbb{E}(L_{sp}\ \cos\ B_{sp}) = \int_0^{\frac{\pi}{2}}\int_0^{\infty} l\ \cos\ \beta f_{sp,L,B}(l,\beta)\ dl\ d\beta$$

denotes the mean of $l\ \cos\ \beta$. (We used here the notation L_{sp} and B_{sp} for the random variables of length and polar angle.) For isotropically oriented fibers of constant length l_0, i.e. the case studied

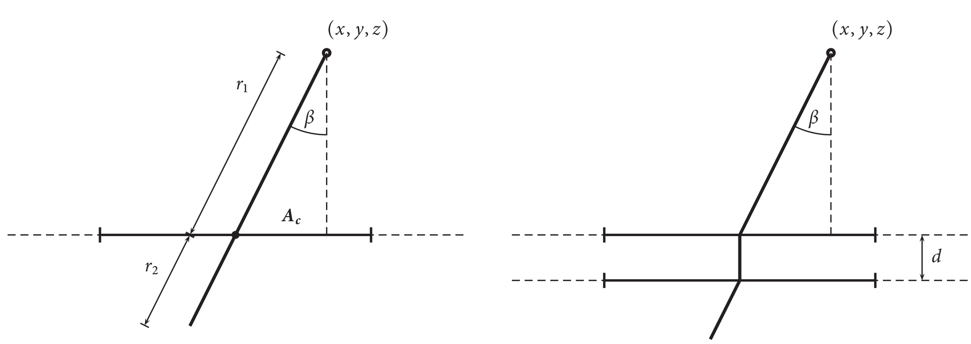

Fig. 1. Left: A fiber intersecting the crack plane A_c having length $l = r_1 + r_2$ and inclining angle β. The reference point of the fiber is denoted by (x, y, z). Right: At crack width d the embedded fiber is being pulled out of the matrix.