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Synthetic turbulence: A wavelet based simulation

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ABSTRACT

The treatment of wind-induced vibrations is an important consideration in the design of civil structures with increasing span-lengths and heights. The turbulence in the atmospheric boundary layer has been treated as Gaussian in conventional stochastic simulation schemes, wherein higher-order statistics have been disregarded. However, experimental evidence points at turbulence being a typical multifractal process, which suggests that the statistics at different scales of atmospheric turbulence are not strictly self-similar but exhibit stronger non-Gaussianity as the length scale decreases. Intermittency characterized by the occasional bursts in the wind velocity leads to non-Gaussianity. Intermittency and its potential impact on wind-induced response have been neglected though. Recent studies have addressed the multifractal property of turbulence in wind with wavelet-basis representations of the log-Normal or log-Poisson models. These schemes offer new insight into the simulation of turbulent wind, but suffer from several significant drawbacks, such as the inappropriate sampling of the wavelet coefficients. To overcome these shortcomings, a new simulation scheme, based on the Haar wavelet representation, is proposed in this study where the exact relation between the wind velocities and the wavelet coefficients is introduced. In addition, the effects of intermittency on the wind-induced response of structures are evaluated for a number of cases. A quasi-steady theory-based assessment indicates that the intermittency results in amplifying extremes of the wind-induced response and exhibits higher impact on relatively more rigid structures. It is likely that intermittency may invoke flow-structure interaction with possible enhancement in load effects at related scales, which may further amplify the extremes.

1. Introduction

Civil structures, such as bridges, buildings, transmission towers and residential structures, are vulnerable to the winds and the associated turbulence. Turbulence has attracted a wide range of studies including the chaotic features, which imply high level of nonlinearity and unpredictability. In general, there are two distinct approaches to describe a turbulent flow: one developed from first principles using the equations of motion, e.g. Navier–Stokes equations, or the probabilistic framework utilizing statistics of turbulent flows based on observations.

Numerical simulation of turbulent wind field using the Navier–Stokes equations is computationally very intensive, especially at high Reynold numbers, whereas statistical approaches can significantly reduce the computational effort by utilizing turbulence statistics. Conventionally, turbulence in the wind field is characterized by the second-order statistics. For example, both the spectral representation and the time series models are based on the second-order statistics with implied assumption of Gaussianity [15,16,21]. However, turbulence has significant higher-order information, which is usually described in terms of velocity increment between two time instants. Experiments have shown that the statistics of the velocity increment depart gradually away from Gaussianity as the time interval decreases [2,5,7]. The K41 theory of turbulence divides the velocity increment into energy-containing, inertial and dissipation sub-ranges [13]. The probability density functions (PDFs) of velocity increments in the inertial sub-range present longer tails than the Gaussian distribution. The non-Gaussianity is highly correlated with the intermittency in turbulent flows. Theoretically, non-Gaussianity features of the PDF reflect the presence of intermittency in the velocity increments and gradients (e.g., [23]). The intermittency has potential effects on wind-induced response because it occurs suddenly with attendant high energy, which may be underestimated by a tacit Gaussian assumption. The higher-order statistics of the wind velocity process can be approximately derived according to their relation to the second-order statistics. For example, both the log-Normal model [14] and the log-Poisson model [19] show high-fidelity for the derivation of higher-order statistics. The log-Poisson model is shown to perform better in She and Waymire [20], and thus it has been used in the present simulation scheme. Indeed, both the log-Normal model and the log-

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Poisson model introduce a multifractal description of wind fluctuations, rendering the simulation process non-trivial. In this context, a wavelet expansion, which is a multi-scale modeling tool, is a preferred basis (e.g., [1,10,12]). In this approach, the sampling of wavelet coefficients from the multifractal statistics is crucial to a high-fidelity simulation of the wind field. In these references initial work in this context was introduced which is further explored and refined here.

This study proposes a new scheme for simulating wind velocity based on the Haar wavelet. This entails several assumptions, i.e., (1) the wind velocity increment in the energy-containing subrange is only affected by the boundary conditions; (2) statistics of the wind velocity increment in the inertial sub-range follow a log-Poisson model for both stationary and non-stationary winds; (3) the wind velocity increment in the energy-dissipation subrange has negligible contribution to wind fluctuations with practical implications, and is therefore disregarded. The second assumption relates to the Kolmogorov's conjecture that within a small time interval the statistics of velocity increment are approximately steady even when the wind velocity is not [13].

The higher-order effects in velocity increments related to intermittency are highlighted here. Typically, the wind-induced response is based on the gust loading factor which involves a peak factor derived under Gaussian assumption. Three major issues, which have been usually ignored, are taken into account in the study to examine their contribution: the quadratic velocity term; the inclusion of relative motion in the formulation of aerodynamic force; the higher-order statistics of velocity fluctuations.

2. Description of the wind velocity process

A wind velocity as a random process can be expressed in terms of the Karhunen–Loeve (K–L) expansion as

$$V(\boldsymbol{\theta}, t) = \sum_{j=0}^{N} \xi_j(\boldsymbol{\theta})\lambda_j\varphi_j(\boldsymbol{\theta}, t) \tag{1}$$

where $V(\boldsymbol{\theta}, t)$ represents a wind velocity indexed by time t; $\boldsymbol{\theta}$ is the vector of the underlying random parameters of the wind velocity process; $\left\{ \xi_j(\boldsymbol{\theta}) \right\}_{j=0}^{N}$ denotes a normalized decomposed random vector with a scaling vector $\left\{ \lambda_j \right\}_{j=0}^{N}$, which also represents the eigenvalue vector of K–L expansion; $\left\{ \varphi_j(\boldsymbol{\theta}, t) \right\}_{j=0}^{N}$ is the corresponding basis.

The Fourier basis is often used in the simulation of a stationary process, whereas a wavelet basis is equally applicable to both stationary and non-stationary processes. Both the spectral representation and the time series (e.g., ARMA) models imply stationarity with the assumption that $\left\{ \xi_j(\boldsymbol{\theta}) \right\}_{j=0}^{N}$ is a Gaussian random vector [15,16,21]. The basis $\left\{ \varphi_j(\boldsymbol{\theta}, t) \right\}_{j=0}^{N}$ can be further expanded in terms of a wavelet basis as (e.g., [8,18])

$$\varphi_j(\boldsymbol{\theta}, t) = \sum_{k=1}^{M} \zeta_{j,k}(\boldsymbol{\theta})\psi_{j,k}(t) \text{ for } j = 0, \ldots, N \tag{2}$$

where $\left\{ \psi_{j,k}(t) \right\}_{j=0,k=1}^{N,M}$ is the wavelet basis; $\left\{ \zeta_{j,k}(\boldsymbol{\theta}) \right\}_{k=1}^{M}$ is the Gaussian random vector. As a result, the wind velocity can be further expressed as

$$\begin{aligned} V(\boldsymbol{\theta}, t) &= \sum_{j=0}^{N} \left\{ \xi_j(\boldsymbol{\theta})\lambda_j \sum_{k=1}^{M} \tilde{\zeta}_{j,k}(\boldsymbol{\theta})\psi_{j,k}(t) \right\} \\ &= \sum_{j=0}^{N} \sum_{k=1}^{M} \zeta_{j,k}(\boldsymbol{\theta}) m_{j,k}\psi_{j,k}(t) \end{aligned} \tag{3}$$

where $\left\{ \zeta_{j,k}(\boldsymbol{\theta}) \right\}_{k=1}^{M}$ denotes a normalized Gaussian random vector with a scaling vector $\left\{ m_{j,k} \right\}_{k=1}^{M}$, which also represents the vector of wavelet coefficients.

Turbulent wind may be treated as a $1/f$ noise whose power spectrum obeys the power law [3]. For example, the power spectrum of a typical turbulent wind has the relation $S(f) \sim f^{-5/3}$ in the inertial range [13]. The power law implies the existence of self-similarity or scale-invariance in turbulence [3], i.e., the PDFs of velocity increments at different length or time scales are similar. Here the velocity increment is defined as $\delta V = V(t) - V(t-\tau)$, in which τ is a time interval. However, further investigation of turbulence structure has shown that the PDFs at different scales are not strictly similar but gradually depart from Gaussian density functions as the length scale decreases. This scale-dependent or multifractal property implies that the turbulent wind is not a Gaussian process (e.g., [4]). Fig. 1 presents the PDFs of velocity increments in various time intervals for the comparison of a measured turbulent wind velocity and the corresponding spectral representation. It shows that, the spectral representation naturally leads to the PDFs at different time intervals to be Gaussian, whereas for the measured turbulent wind, the PDFs present tails longer than the Gaussian. The longer tails in the PDFs are a reflection of the intermittency phenomenon of turbulence. The statistical explanation refers to the theory of large-deviation [7]. Due to the multifractal nature of turbulence, a multi-scale simulation framework like the wavelet representation would be a more appropriate representation of wind velocity fluctuations, which could be utilized for numerical simulation.

2.1. Multifractal property of turbulence

The inertial range of turbulent wind is characterized by two length scales, l_c and l_d, which represent the smallest scales of the energy-containing range and inertial range, respectively. The multifractal property of turbulence can be quantified following two basic parameters, the energy dissipation rate ε_l and the velocity increment δV_l. Here δV_l denotes the velocity increment with time interval l. For the velocity increment within the inertial range, the pth order moments of δV_l and ε_l are both dependent on l [20]

$$\left\langle \delta V_l^p \right\rangle \sim l^{\alpha_p} \tag{4}$$

$$\left\langle \varepsilon_l^p \right\rangle \sim l^{\tau_p} \tag{5}$$

and

$$\alpha_p = \frac{p}{3} + \frac{\tau_p}{3} \tag{6}$$

where $\langle \bullet \rangle$ denotes the expectation operator; τ_p and α_p are both the exponents related to the pth order moment.

2.2. A hierarchical relation of velocity increments

In K41 theory, $\tau_p = 0$, implying that the statistics of ε is scale-invariant. However, for a measured turbulence, the statistics of ε are actually scale-dependent ($\tau_p \neq 0$). Various models have been proposed to model the statistics of δv and ε through the modification of τ_p [14,19]. Among them one of the most promising models may be the log-Poisson model [19,20]. Let l_j denote the length of the jth scale. As j decreases, the scale becomes larger. The log-Poisson model suggests that, within the inertial range, the velocity increments δV_{l_1} and δV_{l_2} have the following hierarchical relation [20]

$$\delta V_{l_2} = W_{l_1 l_2}\delta V_{l_1} = \left[\left(\frac{l_1}{l_2} \right)^{\gamma - 1} \beta^{\kappa_{l_1 l_2}} \right]^{\frac{1}{3}} \delta V_{l_1} \tag{7}$$