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Transient impact response analysis of an elastic–plastic beam

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ABSTRACT

A hybrid, numerical–analytical model is presented to investigate the transient response of a simply supported elastic–plastic beam subjected to impact of a sphere. The model couples a finite difference model based on the Rayleigh's beam theory with a theoretical contact model, named refined Stronge's model. The elastic–plastic transient impact responses of the beam and the wave propagation are solved by the finite difference model. The local elastic–plastic contact behavior is analyzed by the refined Stronge's model. The presented model is verified by the experiments. The comparisons of the numerical results with the experimental results show that the presented model is valid and can predict accurately the transient impact response and impact-induced wave propagation. The comparison of the calculating efficiency with the 3D finite element model shows that the presented model is highly efficient and suitable for parametric study. The effects of the various impact parameters, such as impactor mass, velocity, plasticity and impact location on the impact response, the energy loss and the coefficient of restitution are investigated. It has been found that the impact-induced wave propagation influences significantly the impact force response, the contact time, the energy loss and the coefficient of restitution. A jumping phenomenon of the energy loss and the Newton coefficient of restitution has been found, and it is resulted from the impact-induced wave propagation. The presented model appears to be suitable and convenient for the impact response analysis of elastic–plastic beam, especially for the study of impact-induced wave effects.

1. Introduction

Transient analysis of beam-like structures is significantly important in modeling actual cases [1–3]. As a kind of impact of flexible structures, the impact of beams has long been known to differ from that of compact bodies [1]. Elastic–plastic transient dynamical analysis of beams subjected to impact of projectile involves several complicated phenomena such as local elastic–plastic contact behavior, wave propagation and structural response [1–3]. To reduce the efforts required for analysis for local contact behavior, many theoretical elastic–plastic contact models have been presented independently [3–9], since Marsh [10] and Johnson [4] presented an elastic–plastic contact model to provide theoretically a simply nonlinear contact force-indentation relationship. The theoretical elastic–plastic contact models have been extensively applied to the impact analyses [2,11,12].

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Nomenclature

A	cross-section area of the beam
B	width of the beam
C_n	nth theoretical wave speed
c	damping coefficient per unit length
E	Young's modulus
E^*	effective Young's modulus
E_c	energy loss during compression
E_r	restitution energy during restitution
e	Newton coefficient of restitution
F	contact force
F_y	critical contact force at initial yield
F_m	maximum contact force
f_n	nth theoretical frequency
g	acceleration of gravity
H	height of the beam
I	second moment of cross-section of the beam
J	rotary inertia of the beam per unit length
i	element number in the x direction
j	element number in the z direction
k	number of element to which the impact force is applied
L	length of the beam
M	bending moment
M_s	mass of the sphere
M_b	mass of the sphere and the beam
m	mass of the beam per unit length
N_h	half number of elements along the height of the beam
N_l	number of elements along the length of the beam
n	order of frequency or wave number
$p(x,t)$	distribution of the contact force
Q	lateral shear force
q	instant of time
R	radius of contact body
R^*	effective contact radius
R^{*e}	effective contact radius of unloading
S_n	mark of the nth wave
t	time
v_0	impact velocity
W_c	work done during compression
W_r	work done during restitution
w	displacement
x	horizontal coordinate
x_0	impact position
z	vertical coordinate
Δt	time step
Δx	element size in the x direction
Δz	element size in the z direction
δ	indentation
δ_y	critical yield indentation
δ_p	critical indentation at beginning of full plastic phase
δ_m	maximum indentation
δ_r	permanent indentation
υ	Poisson's ratio
ρ	density
σ	stress
σ^*	stress of previous time
σ_y	yield stress
ε	strain