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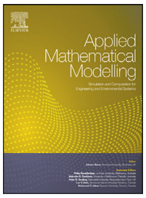

Upper bound analysis of a shape-dependent criterion for closing central rectangular defects during hot rolling

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ABSTRACT

A triangular velocity field for central defect closure is proposed in this paper. With the proposed velocity field, the minimum upper bound power is calculated. Then, the stress state coefficient as a function of defect thickness δ and aspect ratio η is obtained by the upper bound theorem. By applying the limit condition of the stress state coefficient with respect to defect size and letting the defect size be zero, the analytical solution of critical shape factor depending on aspect ratio solely is derived from a quadratic equation. Ultimately, a shape-dependent criterion for closing rectangular defects during hot rolling is established by relating the derived critical shape factor to the actual one. It is shown that as the aspect ratio increases, the critical defect size decreases and the applied energy required for closing central defects increases. The increases in relative reduction and roller radius or the decrease in initial plate thickness are in favor of defect closure. Validation of the present result with available simulation result and a current rolling schedule shows that the present analytical criterion matches well with the simulation result and can be used for optimizing rolling parameters to close central defects. Experimental research in laboratory shows that a defect can be closed well if the actual shape factor reaches or exceeds the corresponding critical shape factor.

1. Introduction

In the center of the continuous casting billet, the existence of various defects, such as voids and internal cavities, is inevitable because of the decrease in gas solubility and volume contraction during the process of solidification. These central defects destroy the continuity of steel materials, which could lead to low qualification rate of ultrasonic inspection for rolled plate. How to eliminate these defects in the subsequent rolling process and consequently guarantee high-quality rolling products is an important task of designing and optimizing rolling processes.

The defect closure behavior has been studied by many investigators, and some important conclusions for eliminating central defects have been proposed through the energy extreme method and experimental research method. Stählberg et al. [1] studied the closure behavior of pre-fabricated pores through experiments, and demonstrated that a large deformation degree produces a high closure extent. Based on several experiments, Wallerö [2] investigated the efficient path to close cen-

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Nomenclature

v_0, v_1	inlet velocity and outlet velocity of rolling workpiece
v, ω	roller linear velocity and angular speed
v_x	horizontal velocity of deformed workpiece
Δv_x	horizontal velocity discontinuity
Δv_{AC}, Δv_{BC}	velocity discontinuities on the discontinuous lines AC and BC respectively
θ	bite angle
α	half of the bite angle
R	roller radius
l	contact length
H, h	initial and final plate thickness
$\bar{h}$	average plate thickness, $\bar{h} = (H + h)/2$
φ_i	angle between discontinuous line AC and horizontal symmetry axis
φ_f	angle between discontinuous line BC and horizontal symmetry axis
λ, δ	half length and half height of a rectangular defect
J^*, $J^*_{\min}$	upper bound power and its minimum value
$\dot{W}_s$, $\dot{W}_\gamma$	shear power and cracking power
J	applied rolling power
M	rolling torque
$\bar{p}$	mean unit rolling pressure
k	yield shear stress
n_σ,n_σ^c	stress state coefficient and its extreme value
η	aspect ratio, $\eta = \lambda/\delta$
δ_c	critical defect thickness
H/R	ratio of initial plate thickness to roller radius
$(H/R)_c$	critical ratio of initial plate thickness to roller radius
Δ, Δ_c	actual and critical shape factors
r, r_c	relative reduction and its critical value

tral pores, and pointed out that the rolling with heavy reduction is good for closing central pores. Wang et al. [3] analyzed the two related procedures for the elimination of the artificial pore in the hot rolling slab, i.e. defect closure and healing, and stated that a bigger roller diameter is in favor of central pore elimination. By the upper bound method, Turczyn [4] established a curved triangle velocity containing a central defect and proposed a critical condition for eliminating the assumed central defect. The present author [5] proposed a criterion for predicting defect closure based on a two-dimensional continuous velocity filed, and the effect of some key rolling parameters, such as the relative reduction and roller radius, on the defect closure has been discussed. However, the effect of defect shape characteristic, described by aspect ratio, has not been considered in the criterion. Kakimoto et al. [6] conducted a compression experiment on the closure of cylindrical voids. Their result shows that the required reduction for closing voids completely depends on the height of the void.

The FE method is an efficient method for simulating the behavior of central voids. By the rigid-plastic FE method, Pietrzyk et al. [7] established a model for analyzing the behavior of internal voids in the rolling plate. It is shown that the temperature difference in the workpiece along the thickness direction affects the closure markedly. By the FE method, the deformation mechanism of internal voids in a sheet during rolling processes was researched by Huang and Chen [8], and the critical condition of thickness reductions for closing the voids completely was also analyzed. Based on the FE method, Chaijaruwanich et al. [9] explored the closure condition by simulating the stress distribution and strain distribution along the thickness direction during hot rolling. It is concluded that when the ratio of the contact arc length to the initial height of the plate is greater than 0.3, pore closure could be guaranteed. Through the FE method, Ji et al. [10,11] investigated the closure behavior of spherical and cylindrical voids under the plane strain condition. Their result indicates that due to the lack of hydrostatic pressure and equivalent strain in the central layer, the void is more difficult to be closed. Li et al. [12] established a FE model containing a central void to simulate the closure behavior, and found that the temperature-difference rolling is more favorable to the closure process, since there are more stresses and strains in the central layer.

Ono et al. [13] investigated the influence of pre-cooling of billet surface on the closure of central defects, and demonstrated that the cooling before the rolling gap has the role of increasing the hydrostatic pressure at the central layer, which benefits defect closure. Chen et al. [14] proposed a three-dimensional model for closing the pores in a cold rolling plate, and pointed out that the pore size and roller size significantly influence the pore closure. Yu et al. [15] established a FE model of inner crack closure considering the roughness of the crack during the healing procedure. It is shown that the introduction of the initial crack roughness is useful in explaining the formulation of residual voids after the forth closure procedure. Deng et al. [16] simulated the closure characteristic of a central defect in a heavy plate during the rough rolling stage. They present the critical shape factor of 0.518 for central defect closure. Based on FE simulation and laboratory exper-