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An enhanced subinterval analysis method for uncertain structural problems

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ABSTRACT

This paper proposes an enhanced subinterval analysis method to predict the bounds of structural response with interval parameters, which could deal with problems with relatively large uncertainties of the parameters. The intervals are first divided into several subintervals, and two expansion routes are then constructed based on the sensitivity analysis. Two subinterval sets are selected according to the expansion points on the routes, and the first order Taylor expansion method is then adopted to complete the subinterval analysis. Based on the selected subinterval sets, the upper and lower bounds of the structural response are further obtained by employing the interval union operation. An adaptive convergence approach is presented to determine the appropriate number of subintervals. Four numerical examples are investigated to demonstrate the validity of the proposed method.

1. Introduction

Due to the existence of environment factors, manufacturing and measurement errors, a variety of uncertainties exist in practical problems associated with the material properties, geometrical characteristics, loads and other parameters. The probabilistic methods have been widely studied to deal with the uncertain problems, in which the uncertain parameters are treated as random variables with precise probabilistic distribution functions [1–3]. However, the complete statistical information is required to define a probabilistic distribution by using the probability approach, which is generally difficult to realize due to the restriction of experiment conditions and cost, especially at the early stage of design. As a result, some non-probabilistic methods such as interval analysis [4–6], convex model [7,8] and fuzzy sets [9] have emerged as beneficial supplements to the conventional probabilistic methods.

Interval analysis based on the set theory is very appropriate for practical engineering problems with uncertain parameters whose lower and upper bounds are only known but information about the probabilistic distribution is missing [1]. Interval arithmetic was proposed to calculate the lower and upper bounds of interval response functions [10,11]. However, one of the major shortcomings is the overestimation in interval computation. Neumaier [5] investigated the hypercube approximation for the united solution set of the interval equations by the Gaussian elimination scheme, which, however, is extremely conservative due to a large number of elimination operations. To overcome the overestimation phenomenon, Wu

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et al. [12,13] developed a Chebyshev interval method which was successfully applied to the dynamic differential problems. However, its computational cost will be very high when more interval variables are involved. By assuming the intervals as uniform distributed variables, the Monte Carlo simulation method (MCS) can be regarded as the simplest approach for interval analysis, which is usually used as a reference method for validating the accuracy of other interval methods [14]. The exact response intervals can be obtained by the vertex method when response functions are monotonic [15]. Based on the matrix perturbation and interval extension theory, the interval perturbation method was proposed by Qiu et al [16]. Wu et al. [17] then employed the interval perturbation method to analyze the dynamic responses of linear structural systems. In addition, Wang et al. [18,19] used the first order Taylor expansion to calculate the structural response interval with measurement data. Gao et al. [20] and Wang et al. [21] carried out the structural analysis with probabilistic and interval mixed uncertainties based on a combination of the first order Taylor-series expansion and matrix perturbation theory. Sevillano et al. [22] presented a modal interval analysis method to estimate damage structural problems with uncertain–but–bounded parameters. The interval analysis methods have also been applied in time-variant problems [23,24], nonlinear structural problems [25,26], acoustic problems [27,28] and other applications [29–34].

Most of the aforementioned methods are only limited to problems that the uncertainty level of the interval variables is relatively small. Thus, theoretically, they cannot be used to solve the interval response functions effectively with a relatively large uncertainty level. To address this important issue, Qiu and Elishakoff [35] proposed a subinterval analysis method to compute the bounds of static displacement responses with a large level of parametric uncertainty. Afterwards, the subinterval analysis method has been obtaining more and more attention. Zhou et al. [36] suggested a subinterval analysis method and also its error estimation technique. Xia and Yu [37] developed a modified sub-interval perturbation finite element method (FEM) to determine the bounds of sound pressure in the 2D acoustic field. Wang and Qiu [38] proposed a first-order subinterval perturbation method to solve the heat conduction problem with uncertainty. The existing subinterval analysis method seems useful for solution of interval response functions when the uncertainty level of the interval variables is relatively large. However, it generally involves a large number of combinations of the subintervals by taking one subinterval from each interval parameter after the intervals are divided into subintervals, and the interval analysis needs to be repeatedly carried out in all possible combinations of subintervals to predict the required response interval. With the increasing of the interval parameters' dimension, the "combination explosion" problem thus will arise, and it generally will lead to an extremely large computational cost. Therefore, it seems necessary to develop a new subinterval analysis approach that not only has a high accuracy for large uncertainty problems but also has an acceptable computational efficiency.

This paper aims to propose an enhanced subinterval analysis method, by which the lower and upper bounds of a structural response with a large uncertainty level can be well predicted. The basis of the approach is analyzing the interval problem by constructing several approximations of the structural responses which are valid over small subintervals. The novel aspect of this paper comes in the way these subintervals are analyzed. In fact, the approximations of the structural response are constructed at a few selected subintervals, thus decreasing the overall numerical cost significantly. The following text consists of three main parts. First, the conventional subinterval analysis method is introduced. Second, an enhanced subinterval analysis method is proposed. Third, four numerical examples are provided to verify the validity of the proposed method.

2. The existing subinterval analysis method

Assuming that only the bounds are known for each uncertain parameter in a practical engineering problem, the uncertain parameter vector $\mathbf{x}$ can then be represented by an interval vector $\mathbf{x}^I$:

$$\mathbf{x}^I = [\underline{\mathbf{x}}_i, \bar{\mathbf{x}}_i] = \left\{x_1^I, x_2^I, \ldots, x_m^I\right\}^T, \ x_i^I = [\underline{x}_i, \bar{x}_i], \quad i = 1, 2, \ldots, m \tag{1}$$

where x_i^I denotes the ith interval parameter; m denotes the dimension of the interval vector $\mathbf{x}^I$; the underline and overline denote the lower and upper bounds of an interval parameter, respectively.

The middle point vector $\mathbf{x}^c$ and radius vector $\mathbf{x}^w$ of the interval vector are defined as:

$$\mathbf{x}^c = \{x_1^c, x_2^c, \ldots, x_m^c\}^T \tag{2}$$

$$\mathbf{x}^w = \{x_1^w, x_2^w, \ldots, x_m^w\}^T \tag{3}$$

where,

$$x_i^c = \frac{\bar{x}_i + \underline{x}_i}{2}, \ i = 1, 2, \ldots, m \tag{4}$$

$$x_i^w = \frac{\bar{x}_i - \underline{x}_i}{2}, \ i = 1, 2, \ldots, m \tag{5}$$

The uncertainty level of an interval number x_i^I is defined:

$$\gamma\left(x_i^I\right) = \frac{x_i^w}{x_i^c} \tag{6}$$