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Time-dependent analysis for a two-processor heterogeneous system with time-varying arrival and service rates

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ABSTRACT

In this paper, we investigate the time-dependent behavior of a two-processor heterogeneous system where the arrival and service rates are allowed to vary with time. We derive an integral equation where the time-dependent probabilities of the two-processor heterogeneous system are expressed in terms of a Volterra equation of the second kind. The effectiveness of our procedure is illustrated with some numerical examples. Finally, a brief comparison is given to show the efficiency and accuracy of the proposed method.

1. Introduction

It is useful to understand queueing systems with time-varying parameters as such systems are commonly seen in queueing theory. Queueing systems with time varying parameters are used to model a wide variety of applications. Some specific cases and applications of these kinds of queues are given in Collings and Stoneman [5], Green and Kolesar [8], Koopman [9], Kolesar [14] and Landauer and Becker [18]. These applications include waiting time for emergency ambulance services, security checkpoints, automatic teller machines, arrival and departure clearance for aircraft at airports, multi-car dispatch of police and many others.

Much effort has been devoted to creating new tools for the analysis of queues with time-varying rates. This is because standard methods for treating time-independent models, cannot be easily generalized to non-stationary cases. Therefore, several authors have attempted to find transient solutions for non-stationary models by introducing novel methods. Some important works on time-independent queues are given in Newell [25] and Keller [13]. They use the approximations and/or perturbation methods to identify several different time ranges where the behavior of the queue is different. Markovian single and multiple server queues with time-varying parameters have been analyzed by many authors (See e.g., [2,10–12,15,20–22,30,32,33] and the references therein). Extra references and outline of numerous mathematical techniques concerning queueing systems, where all the parameters depend on time, are accessible in Massey [24], Schwarz et al. [28] and Stolletz [29].

The essential supposition in concentrating on multi-server queueing systems is that the structure of the servers homogeneous, i.e., the individual service is the same for all servers. In real life, the servers are diverse in their service rates. In

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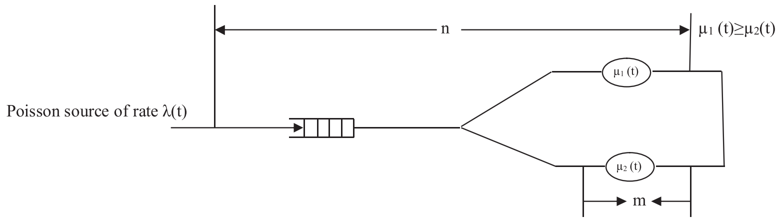

Fig. 1. Model of a two-processor heterogeneous system.

most cases, for the purpose of simplicity, specialists suppose homogeneity to get a simple analysis. Heterogeneous servers are included in the models for some viable situations, such as, nodes in telecommunications networks with links of various amplitudes, nodes in wireless systems serving diverse mobile users, servers shaped with various processors as a consequence of system updates, communications network system supporting communication channels with various transmission rates, multiprogramming computer systems that spools their yield for using a number of printers with various speeds, or scheduling jobs on functionally equivalent processors of a local computer network. In a heterogeneous situation, resources are independent, appropriated, thick, and dynamic. Subsequently, they ought to be viable and booked to make the best possible use of the resources. For more points of interest on this subject, see Chakka and Do [4], Isguder and Kocer [7], Larsen and Agrawala [16] and Lin and Kumar [17].

Onother hand, a multiprocessor system is considered to be one of the most important systems in queueing systems, because it has wide applications in telecommunications, flexible manufacturing systems, reliability and traffic control. Many researchers have studied multiprocessor systems. In Mitrani [23], the steady state probabilities have been obtained for some multiprocessor systems. For related literature, refer to Parthasarathy et al. [26] and Parthasarathy and Sudhesh [27], and the references cited therein.

We consider a multiprocessor system consisting of two types of processors, which for convenience will be referred to as the "main" and the "backup" processors. Each job requires exactly one processor for its execution. When both processors are idle, the main processor is scheduled for service before the backup processor. In Trivedi [31], a similar model with two processors main and backup was presented and discussed for the case of the steady state. In Dharmaraja [6], the transient solution has been obtained for the same system where all its parameters are time-independent. Indeed, there are several papers that dealt with a two-processor heterogeneous system when all parameters are time-independent. For instance, in Ammar [3], the transient behavior of a two-processor heterogeneous system with catastrophes, server failures, and repairs is investigated. Contrary to all the aforementioned works, in Stadje [30] the transition probabilities of $M(t)/M(t)/2$ with homogeneous servers have been discussed when the arrival and service rates are allowed to vary with time.

Consequently, the motivation of the current work is to discuss the transient probabilities for a two-processor heterogeneous system where the arrival and service rates are allowed to vary with time. Using generating functions, the present model can be reduced to solving Volterra integral equations. We can express $P_{n+1,\,1}(t)$, the probability that $n+1$ jobs are in the queue or are being served in terms of a Volterra equation of the second kind. Each of the other transient probabilities is expressed in terms of integral equations in $P_{11}(t)$.

This article is organized as follows: Section 2 gives a description of the suggested model and discusses the main assumptions considered. Section 3, presents a detailed analysis of the main results by using the generating function technique to obtain the transient probabilities in terms of a Volterra equation of the second kind. Section 4 contains numerical results, in addition to a comparison with the Runge–Kutta method, a simulation and the results in Dharmaraja [6] and Zhang and Coyle [33]. Finally, the main conclusion of the paper is presented in Section 5.

2. Model descriptions

Consider a computer system consisting of two processors, a main processor, and a backup processor. A detailed description of the system is as follows:

(1) Jobs arrive at the system according to a Poisson process with an arrival rate $\lambda(t)$. Service is exponentially distributed, and two servers provide heterogeneous service rates $\mu_1(t)$ and $\mu_2(t)$ such that $\mu_2(t) \le \mu_1(t)$.The queueing structure is shown in Fig. 1. The state-transition diagram of the system is given in Fig. 2.
(2) Each job needs only one server for service and the jobs select the servers on the basis of fastest server first (FSF).
(3) Let $Q_M(t)$ be the number of customers in this queue (including the one in service) and $Q_B(t)\,(=0,\,1)$ denote whether there is anyone being serviced by the backup processor. Clearly, $Q_M(t)$ can take the values $\{0,\ 1,\ 2,\ \ldots\}$, and whenever $Q_M(t) > 1,\quad Q_B(t)=1$. The total number of customers in the system is obtained by $Q_M(t)+Q_B(t)$. Let

$$P_{nm}(t) = P[\,Q_M(t) = n, Q_B(t) = m].$$

From these assumptions, the forward equations can be written as follows:

$$P'_{00}(t) = -\lambda(t)P_{00}(t) + \mu_1(t)P_{10}(t) + \mu_2(t)P_{01}(t), \tag{2.1}$$