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Particle methods for multi-group pedestrian flow

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ABSTRACT

We consider a multi-group microscopic model for pedestrian flow describing the behaviour of large groups. It is based on an interacting particle system coupled to an eikonal equation. Hydrodynamic multi-group models are derived from the underlying particle system as well as scalar multi-group models. The eikonal equation is used to compute optimal paths for the pedestrians. Particle methods are used to solve the equations on all levels of the hierarchy. Numerical test cases are investigated and the models and, in particular, the resulting evacuation times are compared for a wide range of different parameters.

1. Introduction

Pedestrian flow modelling has attracted the interest of a large number of scientists from different research fields, as well as planners and designers. While planning the architecture of buildings one might be interested in how people move around their intended design so that shops, entrances, corridors, emergency exits and seating can be placed in useful locations. Pedestrian models are helpful in improving efficiency and safety in public places such as airport terminals, train stations, theatres and shopping malls. They are not only used as a tool for understanding pedestrian dynamics at public places, but also support transportation planners or managers to design timetables.

A large number of models for pedestrian flow have appeared on different levels of description in recent years. The microscopic (individual-based) level models based on Newton type equations as well as vision-based models or cellular automata models and agent-based models have been developed, see Refs. [1–8]. Hydrodynamic pedestrian flow equations involving equation for density and mean velocity of the flow are derived in Refs. [9–11]. Modeling of pedestrian flow with scalar conservation laws coupled to the solution of the eikonal equation has been presented and investigated in Refs. [12–17]. Pros and Cons of these models have been discussed in various reviews, we refer to [18–20] for a detailed discussion of the different approaches.

The modelling of pedestrian behaviour in a real-world environment is a complex problem. For example, a majority of the people in a crowd are moving in groups and social interactions can greatly influence crowd behaviour. Most of the models mentioned above treat pedestrians as individual agents and neglect the group dynamics among them. The influence of group dynamics on the behaviour of pedestrians and the differences between the behaviour of pedestrians walking in groups and single pedestrians have been presented in several recent works. We refer to [4,21–27]. In these works experimental studies as well as numerical experiments are presented.

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In this work, we closely follow a procedure for interacting particle systems used, for example, in the description of coherent motion of animal groups such as schools of fish, flocks of birds or swarms of insects, see Refs. [28,29]. It has been applied to pedestrian flow modelling in Ref. [11]. There, a classical microscopic social force model for pedestrians [1] has been extended with an optimal path computation as for example in Ref. [14].

One main objective of the present paper is to include multi-group behaviour and the impact of group dynamics, addressing in particular larger groups in a pedestrian crowd, into the set-up developed in [11]. We extend the model developed there to the description of multi-group pedestrian flows using a multi-phase approach. The dependence of the solutions on the level of attraction between the group members and the relaxation time towards the desired optimal path field is investigated and discussed. As a general result we observe an increase in evacuation time by increasing the attraction between the group members. The second objective of the paper is to show the usefulness of using a unified approach for the numerical simulation of pedestrian models on microscopic and macroscopic scales. We use, as in Ref. [11], particle methods on the microscopic and macroscopic level of the model hierarchy. These methods are straightforward for microscopic equations. In case of the macroscopic equations particle methods are based on a Lagrangian formulation of these equations and particles are used as grid points. A numerical comparison of different numerical approaches in microscopic and macroscopic situations is presented. Moreover, we note that the method presented here is easily extended to more complicated ”real” life situations, since the numerical implementation is based on a mesh-free fluid dynamic code for complex geometries.

The paper is organized in the following way: in Section 2 the hierarchy of multi-group pedestrian models is presented. Section 3 contains a description of the particle methods used in the simulations. Section 4 contains the numerical results. We consider an evacuation problem. A comparison of the solutions of the macroscopic equations is presented for different parameters together with a comparison of the associated evacuation times. Finally, Section 5 concludes the work.

2. Multi-group pedestrian flow models

In this section, we start with a multi-group microscopic model for pedestrian flow using a microscopic social force model and a Hughes-type model including the solution of the eikonal equation. We proceed by deriving multi-group hydrodynamic and scalar models from the microscopic model.

2.1. The microscopic multi-group model

We consider a microscopic social force model for pedestrian flow including an optimal path computation. For references, see for example Refs. [1,14]. For N pedestrians divided into M groups, we obtain a two-dimensional interacting particle system with locations $x_i^{(k)} \in \mathbb{R}^2$, and velocity $v_i^{(k)} \in \mathbb{R}^2$. Here, the index $i = 1, \dots, N$ is used to number all pedestrians, the index $k = 1, \dots, M$ denotes the group to which the pedestrian belongs. $S^{(k)}$ denotes the set of all i which are in group k and N_k denotes the number of pedestrians in group k with $N = \sum_{l=1}^{M} N_l$. The equations of motion are

$$\begin{aligned} \frac{dx_i^{(k)}}{dt} &= v_i^{(k)} \\ \frac{dv_i^{(k)}}{dt} &= -\sum_{l=1}^{M} \sum_{j \in S^{(l)}} \nabla_{x_i} U^{(k,l)}(\mid x_i^{(k)} - x_j^{(l)} \mid) + G^{(k)}(x_i^{(k)}, v_i^{(k)}, \rho^N(x_i^{(k)})), \end{aligned} \tag{1}$$

where $U^{(k,\, l)}$ is an interaction potential denoting the interaction between members of groups k and l. A common choice is the Morse potential

$$U^{(k,l)}(r) = -C_a e^{-r/l_a} + C_r e^{-r/l_r}. \tag{2}$$

Here, C_a, C_r are attractive and repulsive strengths and l_a, l_r are their respective length scales. These constants depend on the groups k and l under consideration. Similarly, one could use potentials given by poynomial or rational functions. An attractive interaction force acts only between members of the same group. The repulsive force acts between all pedestrians. The acceleration towards the desired direction is given by

$$G^{(k)}(x, v, \rho^N) = \frac{1}{T}\left(-V^{(k)}(\rho^N)\frac{\nabla\Phi^{(k)}(x)}{\|\nabla\Phi^{(k)}(x)\|} - v\right). \tag{3}$$

Moreover, ρ^N is given by

$$\rho^N(x) = \frac{1}{N}\sum_{l=1}^{M}\sum_{j \in S^{(l)}} \delta_S(x - x_j^{(l)}),$$

where δ_s is a smoothed version of the δ-distribution with $\int \delta_S(x)dx = 1$ Finally, $\Phi^{(k)}$ is given by the solution of the eikonal equation

$$V^{(k)}(\rho^N(x))\|\nabla\Phi^{(k)}\| - 1 = 0.$$