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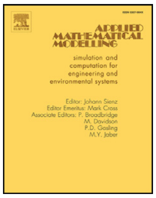

Macroscopic modeling of pedestrian flow based on a second-order predictive dynamic model

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ABSTRACT

This work presents a second-order predictive dynamic model for pedestrian flow to investigate movement patterns and non-equilibrium phenomena in pedestrian traffic. This model is described as a system of nonlinear hyperbolic conservation laws with relaxation under the hypothesis that a group of pedestrians are regarded as a continuous anisotropic medium. The desired or preferred walking direction of pedestrians is assumed to minimize the total actual walking cost based on predictive traffic conditions, which satisfies the predictive dynamic user-equilibrium assignment. To solve this model, a cell-centered finite volume method for hyperbolic conservation laws coupled with a self-adaptive method of successive averages for an arisen discrete fixed point problem is adopted. The proposed model and algorithm are validated by comparing the results carried out by the model with experimental observations under non-congested conditions. Numerical examples are designed to investigate macroscopic features and path-choice behaviors of pedestrian flow. Numerical results indicate that the proposed model is able to reproduce some complex nonlinear phenomena in pedestrian traffic, such as the formation of congestions and stop-and-go waves.

1. Introduction

In any large crowd, people could be injured and even lose lives because of the dynamics of the crowd's behavior [1]. The importance of understanding pedestrian crowd dynamics and simulating behaviors and movements of the crowds has regained a significant level of interest. There are typically three types of crowd motion simulation models based on different descriptive details: microscopic models (like social force models [2–4], cellular automaton models [5–8] and agent-based models [9–11]), mesoscopic (kinetic) models [12,13] and macroscopic models [14–20]. None of the three types of models is completely satisfactory since various technical and conceptual advantages and drawbacks are linked to modeling at each scale [13]. In the case of a large crowd, macroscopic characteristics of pedestrian flow (e.g., speed, density and flow) are of

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prime interest. Therefore, to better understand macroscopic features of pedestrian crowd motion in especially large crowds, pedestrian dynamic models presented at a macroscopic level are necessary and significative to study this topic due to relatively lower computational complexity and fewer model parameters. Basically, macroscopic pedestrian flow models can be derived from the underlying mesoscopic (kinetic) description for active particles [13].

Based on the analogy between the traffic flow and fluid flow, Lighthill and Whitham [21] first proposed the well-known first-order LWR model. Since then many other methods and models have been developed based on the mathematical modeling of crowd dynamics within the framework of continuum mechanics [14–20,22–27]. In these models, a crowd of pedestrians is generally treated as a continuum, provided the characteristic distance scale between pedestrians is much less than the characteristic distance scale of the region where the pedestrians move. Analogous to macroscopic vehicular traffic flow models, macroscopic pedestrian flow models are classified as first-order models that only involve the mass conservation equation and second-order models or dynamic models that involves both the mass conservation equation and linear momentum equation. Hughes [14] presented a continuum theory for the first-order pedestrian flow and established equations of motion that govern both single and multiple pedestrian types caused by differences among pedestrian walking patterns. Huang et al. [18] revisited Hughes' model and verified that the developed model can capture the formation of shockwaves. However, due to the assumption that the traffic flow is always in an equilibrium state, the first-order models are not capable of explaining other complex phenomena, e.g., clogging at bottlenecks and localized clusters or stop-and-go waves [22,26]. For instance, qualitatively similar stop-and-go traffic that corresponds to regions with relatively high density and relatively low speed which propagate backward the flow has been experimentally observed in high density crowd scenes [28–30]. At even higher densities, a sudden transition from stop-and-go waves to "crowd turbulence" could arise before crowd disasters [31,32]. To resolve this problem in the first-order models, several second-order models [17,22,26,33,34] have been proposed. In these models, the equilibrium speed-density relationship is replaced by a dynamic equation with respect to the average velocity of pedestrian flow. A typical model is the one presented by Bellomo and Dogbé [17] which consists of the equations of conservation of mass and equilibrium of linear momentum involving an acceleration term. However, the desired direction of motion in this model is fixed at any point of the facility and does not change with the time-varying traffic conditions. Therefore, it can't quite express the real path-choice behavior of pedestrians.

For the issue mentioned above, in this paper a second-order predictive dynamic model for pedestrian flow is presented to investigate macroscopic characteristics of pedestrian movement and some non-equilibrium phenomena, such as the experimentally observed stop-and-go waves [29,30]. This model is described as a two-dimensional (2D) hyperbolic system of nonlinear conservation laws with source terms under the hypothesis that a group of pedestrians move like a continuous anisotropic medium. In the model, the pedestrian path-choice strategy is determined based on predictive traffic information gained through experience. Specifically, the desired or intended walking direction of pedestrians is to minimize the total actual walking cost from/at the current position/time to the destination based on predictive travel cost information gained through experience, which satisfies the predictive dynamic user-equilibrium (PDUE) assignment [35,36]. A linear stability analysis of the presented model shows that non-equilibrium phase transitions, such as the transition from stable to unstable flow, can be described by this model. A numerical method used to solve the predictive dynamic model is designed as a cell-centered finite volume method coupled with a self-adaptive method of successive averages (MSA) for an arisen fixed point problem. To test the validity of the proposed model and algorithm, we compare the results carried out by the model with experimental observations. The proposed model and algorithm are also applied to study crowd movement in a 2D continuous walking facility scattered with a square obstruction to demonstrate their applicability and effectiveness.

The outline of this paper is as follows. In the next section, the mathematical formulation for the movement of pedestrians is described in detail. Section 3 gives a numerical algorithm for the model. The numerical results are presented in Section 4. Finally, some concluding remarks are given in Section 5.

2. Problem formulation

We regard a group of pedestrians walking in a domain denoted by Ω as a compressible continuum fluid medium. The boundary of Ω consists of inflow boundary Γ_i, outflow boundary Γ_o, and solid wall boundary Γ_w. $T = [0, t_{end}]$ (in s) is the modeling period. For the sake of simplicity, t_{end} is considered to be fixed and is large enough to make sure that all pedestrians can leave the modeling domain Ω within the modeling period T. In the context of continuum mechanics, the dynamic model for pedestrian flow, which is composed of equations of conservation of mass and equilibrium of linear momentum [17], is written as follows.

$$\begin{cases} \dfrac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \\ \dfrac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = \mathbf{F}(\rho, \mathbf{v}), \end{cases} \tag{1}$$

where $\rho = \rho(x, y, t)$ (in ped/m^2) and $\mathbf{v} = (u, v)$ (in m/s) denote the density and velocity of pedestrian flow, respectively. Here, the force $\mathbf{F} = [F_x, F_y]$ in Eq. (1) models the local acceleration and characterizes the internal driving force or motivation of pedestrian flow.

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