Applied Mathematical Modelling 40 (2016) 9821–9834

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Numerical modeling of free surface flow in hydraulic structures using Smoothed Particle Hydrodynamics

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ARTICLE INFO

Article history:
Received 23 March 2015
Revised 11 June 2016
Accepted 16 June 2016
Available online 28 June 2016

Keywords:
Smoothed Particle Hydrodynamics (SPH)
Orifice flow
Sharp crested weir
Nappe trajectory
Scale discharge relationship
Inlet and outlet boundary condition

ABSTRACT

A meshless particle numerical model based on the Smoothed Particle Hydrodynamics (SPH) is developed and evaluated for two-dimensional simulation of free-surface flows passing through an orifice and over a sharp crested weir. A practical technique is introduced making the model capable of handling the inlet and outlet boundaries. This technique is able to impose known-inflow flux (as in sub-critical open channel flow) at the inlet boundary, while the inlet pressure/depth is automatically adjusted (without imposing any unwanted boundary reflection and pressure fluctuation). Solution stabilizer techniques, such as XSPH and artificial viscosity, are also employed to increase the accuracy and stability of the model, and their effects on the results are evaluated. The developed model proves to be able to reproduce the free surface complexity with good accuracy. A 3-D finite volume mesh-based model is also employed to verify the result of the present SPH model, showing quite compatible pressure and velocity fields.

1. Introduction

Weirs and orifices are two types of hydraulic structures that are commonly used as flow measurement and regulating structures. These structures regulate the flow and act as a controller, providing a unique relationship between the upstream head and the flow discharge [1]. Many research works have been conducted to study the flow characteristics passing through these hydraulic structures. Several experiments were conducted to study the flow issuing from an orifice such as those by Medaugh et al. [2], Judd et al. [3], Lienhard [4] and Chanson et al. [5]. These researches also aimed to determine discharge coefficient (characterizing the head and discharge relationship) in different conditions. Extensive experimental studies are also carried out on the discharge relationship of sharp crested weirs including those by Rehbock [6], Rouse [7], Kindsvater and Carter [8], Ramamurthy et al. [9], Swamee [10], Brater et al. [11] and Johnson [12]. Flow over sharp crested weirs has also been studied analytically by Strelkoff [13].

From the numerical point of view, majority of the past numerical models used for simulation of flow regulating structures (e.g., [14] and [15]) have been based on Eulerian mesh-based methods. However, due to the rapidly varied flow over the weir and large deformation of the nappe trajectory downstream of these structures, the Eulerian mesh-based methods require complicated interface tracking techniques such as Volume of Fluid (VOF) [14] or Marker and Cell (MAC). Such computationally intensive techniques are known to have difficulties in dealing with the sharp interfaces, and to be associated with numerical diffusion [16].

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http://dx.doi.org/10.1016/j.apm.2016.06.032

In recent years, newer generation of numerical methods, namely meshless particle (Lagrangian) methods, have become a serious alternative to the Eulerian methods especially for cases with large interfacial deformations and fragmentations. Smoothed Particle Hydrodynamics (SPH) [17] is one of the most popular meshless particle methods. The ability of the SPH method in simulation of the highly deformed free-surfaces, makes it a strong tool for modeling the flow over weirs and orifices. A major advantage of the SPH model over Eulerian methods is its ability to capture very complex interfaces without any interfacial tracking treatment [18].

SPH was originally invented to simulate the astrophysical problems [17]. Since then SPH has been extended to handle many types of fluid mechanics problems including free surface flows such as: dam break flow (e.g., [19–22]), Newtonian and non- Newtonian free surface flow [23], fluid–structure interaction (e.g., [24,25]) and breaking waves ([26,27]). Ferrari [28] simulated flow over weirs using a high-performance SPH method.

WCSPH and ISPH are two different approaches for handling the incompressibility in SPH models. The first approach considers the flow as weakly (slightly) compressible so that the pressure field can be explicitly calculated from the density field using an appropriate equation of state. In the majority of the past SPH applications, this approach has been employed and proven it capabilities, for example for gravity currents [29], wave propagation and run-up ([30,31]), wave breaking and post-breaking [32], hydraulic jump [33], hydraulic structures [34] and liquid sloshing dynamics [35]. Alternatively, a fully incompressible SPH (ISPH) method was proposed by Cummins and Rudman [36] in which a two-step projection procedure was implemented to satisfy the incompressibility constraint through solving a Poisson equation for pressure. WCSPH and ISPH have been evaluated and compared by various researchers (e.g., [18,37,20]).

In the standard SPH method, the unity condition of the kernel function (which states that the numerical integration of the applied kernel function must be equal to 1) is not exactly satisfied for a system of arbitrary scattered discrete particles. Some studies (e.g., [38]) proposed a corrected version of smoothed particle hydrodynamic, CSPH, in which SPH kernel is multiplied to guarantee the partition of unity condition. Staroszczyk [39] applied CSPH to simulate dam break flow and compared its results with standard SPH. He also simulated propagation of solitary wave in shallow water by CSPH [39]. Jiang et al. [40] employed CSPH to simulate unsteady viscoelastic flows problems and concluded that the CSPH method leads to more accurate results especially near the boundaries, although it is more computationally intensive.

Dealing with solid boundaries as well as inlet and outlets, is challenging for particle methods such as SPH. Researchers have suggested different approaches to treat the solid boundaries such as using some so-called virtual particles to compensate the near -boundary density deficiency and prevent unphysical penetration of fluid particles into the solid boundaries. Since its original development, the Standard SPH method has been improved by many researchers [41]. These improvements have increased the stability and accuracy of the model by various techniques such as correcting particle velocity by considering the neighbor particles velocities, XSPH, [42] and adding artificial viscosity.

In the present paper, the WCSPH method is developed to simulate the flow through two important flow regulating structures, the orifice and the sharp crested weir. Latest advances in the SPH method, are employed in the present work. In addition, a simple yet accurate technique is introduced for enforcing inlet boundary condition with a known inflow discharge, which is a common boundary condition of subcritical open channel flows.

Further validations are also provided, comparing the SPH results with those from an Eulerian mesh-based method (i.e. a Finite Volume VOF-based model). The results of this study are expected to evaluate the capabilities of SPH method dealing with flows through various regulating structures. It can also provide a springboard for better understanding of the complex flow behavior involved in the examined regulating structures.

2. Governing equations and numerical method

The SPH method represents the continuum with a set of discrete elements referred as particles. The formulation of the SPH method is based on the integral representation of physical quantities, where the physical quantity of a particle is approximated by summing up the relevant values of its neighboring particles using a kernel function (also called smoothing or weight function). The SPH approximations of an arbitrary quantity A (at the position $\mathbf{r}$) and its gradient are written as [43]:

$$\langle A(\mathbf{r})\rangle = \sum_j m_j \frac{A_j}{\rho_j} W\left(\left|\mathbf{r}-\mathbf{r_j}\right|, h\right), \tag{1}$$

$$\langle \nabla A(\mathbf{r})\rangle = \sum_j m_j \frac{A_j}{\rho_j} \nabla W\left(\left|\mathbf{r}-\mathbf{r}_j\right|, h\right), \tag{2}$$

where the operator $\langle\rangle$ denotes for kernel smoothing, the subscript j denotes the number corresponding to the neighboring particle j, m is the mass of particle, $\mathbf{r}$ is the position vector, W is the kernel function, and h is the smoothing length. The density of a particle of interest, i, is obtained by summing the contributions from the neighboring particles, j, as [44]:

$$\rho_i = \sum_j m_j W\left(\left|\mathbf{r}_i-\mathbf{r}_j\right|, h\right). \tag{3}$$