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Train timetabling for an urban rail transit line using a Lagrangian relaxation approach

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A B S T R A C T

Delivering efficient transit services to users is the main objective of public transportation systems. Thus, rail transit systems seek to schedule train services in order to avoid passenger congestion and to minimize the waiting times for passengers. In this study, we present a path-indexed nonlinear formulation of the train timetabling problem for an urban railway system with the objective of minimizing the average waiting time per passenger subject to capacity and resource constraints. The number of planned train services is limited, so the main decisions involved in this scheduling problem are the optimal departure times for all the trains running on the network. A Lagrangian relaxation approach is proposed where the vehicle circulation constraints are relaxed, so the problem can be decomposed into a number of sub-problems for each path. We tested the proposed approach using realistic examples suggested by the Tehran sub-urban railway administration in Iran. The results obtained proved that the strength of the vehicle circulation constraint was dominant. The Lagrangian relaxation algorithm could find optimal solutions for large-scale problems within a reasonable run-time compared with traditional methods using commercial solvers, thereby suggesting the high potential of the proposed solution approach for a metro system.

1. Introduction

Providing cost-effective and efficient transit services to users is the main role of public transportation systems. In this context, a cost-effective operation plan is achievable by optimizing the departure times of vehicles with respect to the fleet capacity and resource constraints. In particular, inefficient transport systems lead to extra traveling times for passengers. A common measure of quality for a public transportation service is the average waiting time (AWT) for passengers. Issues regarding estimating and minimizing the AWT are major concerns for academic research. Thus, Welding [1] proposed an approximation of the AWT function for a transit line where the arrivals of vehicles and passengers at stations were stochastic. Recently, Amin-Naseri and Baradaran [2] proposed improved exact formulations for estimating the AWT at a bus stop with dependent and independent headways. A discrete-event simulation model was developed to verify the accuracy of the proposed formulations.

The train timetabling problem (TTP) for an urban rail transit system involves the determination of the arrival and departure times for each train at each station [3]. A very common method for minimizing the AWT of passengers is applying

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an even-headway timetable. In even-headway timetables, vehicles arrive and depart at constant intervals. This type of operation has been implemented in various rail systems, because of the simplicity and the acceptable service that it provides. However, the rail managers offer only a limited number of train services during the daily operation period because of operational and maintenance costs. In a station-dependent demand situation, an even-headway schedule may result in extended passenger waiting times due to the train capacity constraint, or inefficient vehicle circulation. Hence, the efficient planning of train services should decrease the AWT. The design of a demand-oriented timetable has received much attention in previous studies. Thus, Chierici et al. [4] proposed demand-dependent optimization for regular train timetables with the objective of maximizing the total demand captured by the vehicles. They studied the mutual effect of the quality of the timetable and the transport demand captured by the railway. A mixed-integer nonlinear formulation was presented for the problem while a branch-and-bound and heuristic algorithm was proposed to solve real instances for a regional network in North-western Italy. Albrecht [5] proposed a two-level approach to deal with the TTP for a demand-oriented service on suburban railways, where the optimal train frequency on each line and the capacity of the trains are determined at the first level, and the optimization model generates detailed train timetables at the second level. It has been proved that the demand-driven train service timetable design problem with a capacity constraint is NP-hard [6].

Niu and Zhou [3] developed a nonlinear optimization model for the passenger TTP in an urban rail system subject to resource constraints and dynamic demand. Local improvement and dynamic programming approaches were proposed to solve the TTP optimally for single station cases. A genetic algorithm was presented based on a novel binary coding scheme to solve the multi-station cases. Wang et al. [7] proposed a real-time train scheduling approach for minimizing the passenger waiting time and the passenger traveling time, where they assumed that the passenger arrival rate is static and they formulated the problem as a nonlinear non-convex programming model. The model was solved using a sequential quadratic programming approach to obtain the optimal departure times, running times, and dwell times of trains. They concluded that the optimal headways are affected by the passenger demands at stations. Upadhyay and Bolia [8] proposed a deterministic optimization model for the freight train scheduling problem with given demands over a planning horizon. The objectives were improving the service quality and maximizing profits by reducing empty movements subject to capacity and demand constraints. Hassannayebi et al. [9] employed a two-stage simulation optimization approach with an embedded genetic algorithm in order to minimize the AWTs of passengers in a metro system. Barrena et al. [10] developed time-indexed formulations for the TTP with dynamic demand to minimize the passenger AWT. The train capacity constraints were neglected in the formulation and the TTP for a double-track rail was decomposed into two independent problems. A branch-and-cut algorithm was proposed to solve all the formulations. Recently, Zhu et al. [11] developed a mathematical model for scheduling trains at the start terminal in order to minimize the total passenger waiting time and operating costs subject to station and train capacity constraints. A discrete-event simulation model was employed to simulate the movements of trains and passenger flows. In addition, a genetic algorithm was employed to construct a near-optimal timetable.

Scientific studies have also considered the train rescheduling problem in rail transport services. Thus, Wong et al. [12] addressed the transit synchronization problem with the aim of designing a timetable with the minimal transfer waiting time for all passengers. Thus, a novel mixed-integer linear programming model was developed to consider the turnaround times at terminals and the safety headways at intermediate stations. Xu et al. [13] evaluated three operational strategies, i.e., increasing the train frequency, improving the train capacity, and employing express trains on subway lines. They assumed that the passenger arrival rates and alighting ratios were triangular fuzzy variables, and four assessment indices were considered: the loading ratio, passenger volume, energy consumption, and congestion degree. Xu et al. [14] investigated the train scheduling problem with variable velocity to minimize the ratio of the total delay time and the total free traveling time. They suggested an integrated optimization method, which included an improved travel-advance strategy and a genetic algorithm, to construct an efficient timetable. Kang et al. [15] developed a rescheduling model for an urban rail transit network to consider train delays, where the objective functions included the average transfer time and the deviation from the initial schedule.

Different exact methods have been proposed for railway optimization problems and Cacchiani et al. [16] provided a comprehensive review of this topic. Zhou and Zhong [17] presented branch-and-bound algorithms with improved lower bounds in order to solve the single-track TTP. Min et al. [18] applied a column generation-based approach to centralized train-conflict resolution in a metropolitan rail network.

Castillo et al. [19] proposed an exact algorithm where a bi-section rule is employed to generate tight upper bounds for the objective function with the aim of solving the TTP for a general rail network. Yaghini et al. [20] proposed an improved local branching algorithm for train formation planning. Lin and Kwan [21] developed a branch-and-price method to solve the train unit scheduling problem. Methods based on Lagrangian relaxation have been employed widely for solving train scheduling problems (e.g., see [22–28]). Brännlund et al. [24] presented a novel optimization approach for scheduling a set of trains to find a profit-maximizing timetable subject to track capacity constraints. They used a Lagrangian relaxation solution approach, where the track capacity constraints were relaxed in order to separate the problem into one dynamic program for each train. The approach was tested using a realistic example, which comprised 18 passenger trains, eight freight trains, and 17 stations, as suggested by the Swedish National Railway Administration. Fischer et al. [27] studied the TTP for a rail network and employed a Lagrangian relaxation method integrated with a cutting plane approach. Cacchiani et al. [25] proposed a modified Lagrangian optimization method, which is capable of dealing with robustness, and they applied it to the aperiodic TTP on a rail corridor. Meng and Zhou [29] proposed a novel integer programming formulation for the compound train rerouting and rescheduling problem in complex rail networks. With the aim of obtaining a new