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Finite element simulation of a gradient elastic half-space subjected to thermal shock on the boundary

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ABSTRACT

The influence of the microstructure on the macroscopical behavior of complex materials is disclosed under thermal shock conditions. The thermal shock response of an elastic half-space subjected to convective heat transfer at its free surface from a fluid undergoing a sudden change of its temperature is investigated within the context of the generalized continuum theory of gradient thermoelasticity. This theory is employed to model effectively the material microstructure. This is a demanding initial boundary value problem which is solved numerically using a higher-order finite element procedure. Simulations have been performed for different values of the microstructural parameters showing that within the gradient material the thermoelastic pulses are found to be dispersive and smoother than those within a classical elastic solid, for which the solution is retrieved as a special case. Energy type stability estimates for the weak solution have been obtained for both the fully and weakly coupled thermoelastic systems. The convergence characteristics of the proposed finite element schemes have been verified by several numerical experiments. In addition to the direct applicative significance of the obtained results, our solution serves as a useful benchmark for modeling more complicated problems within the framework of gradient thermoelasticity.

1. Introduction

It is well-known that the material microstructure influences the macroscopical behavior of complex solids, such as composites, cellular materials, and ceramics. Classical continuum theories do not incorporate internal length-scales and therefore cannot capture the pertinent scale effects that are associated with the underlying material microstructure. To this purpose, various generalized (enhanced) continuum theories (for a comprehensive review, see [1]) have been proposed, enriching the classical description with additional material length scales and, thus, extending the range of applicability of the 'continuum' concept in an effort to bridge the gap between classical continuum theories and atomic-lattice theories. These models have also been derived from the theoretical identification of homogeneous materials equivalent to composites with heterogeneous classic phases [2–5] and from experimental testing at small scales [6–8].

In the last decade, the study of microstructured materials through enhanced continuum theories has been significantly boosted by recent advances in the fields of nanomechanics, micromachining, and bioengineering. The use of such theories allows a more accurate description of the mechanical response of high performance microstructured materials for instance, in problems where high strain/stress gradients emerge [9–12] or when instability phenomena are involved [13–15].

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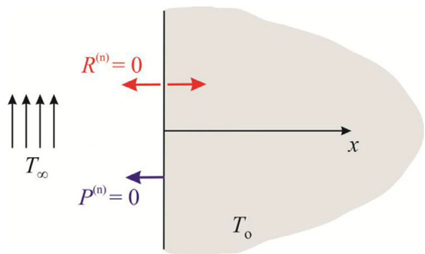

Fig. 1. Gradient elastic half-space subjected to convective heat transfer with a surrounding fluid at its free boundary.

One of the most effective generalized continuum theories has proved to be the theory of gradient elasticity, also known as dipolar gradient elasticity or grade-two theory [16,17]. According to the gradient elasticity theory, the material points inside a continuum can be visualized as micro-continuum with their own internal displacement field described in reference to a local coordinate system. Assuming enough regularity of the deformation process, the internal displacement field of each point can be expanded in Taylor series. If only the linear terms of these expansions are retained, the dipolar theory is obtained. The continuum under consideration consists of structural units (micro-media) in the form of cubes with edge length, which is an inherent length characteristic of the material structure (e.g. grain size). The presence of this length parameter, in turn, implies that the gradient elasticity theory encompasses the analytical possibility of size effects, which are absent in the classical theory. The physical relevance of the characteristic material length scales as introduced through gradient type theories has been the subject of numerous experimental studies. In particular, atomistic calculations and experiments indicate that for most metals, the characteristic internal length is of the order of the lattice parameter [4,18]. However, foam and cellular materials exhibit a characteristic length that is comparable to the average cell size, whereas in laminates is of the order of the laminate thickness [2,7,8].

In recent years, the thermoelastic behavior of complex microstructured materials has attracted considerable attention since their high performance properties are closely related to their reliability under changing thermal conditions. Various gradient type models have been developed in order to describe the thermomechanical response of such microstructured continua [19–24]. In several of these models non-local phenomena in the time dependence of the fields have been also considered, leading to non-Fourier heat transfer models like the Vernotte–Cattaneo model [25,26]. The generalized Green–Lindsay model presented in [23], in a reduced form corresponding to the consideration of classical Fourier heat diffusion, will be the subject of the present analysis. This particular model will be employed for the study of the response of a gradient elastic half-space subjected to thermal shock on its boundary. The thermal shock is induced by convective heat transfer with a surrounding medium that undergoes a sudden change in its temperature (Fig. 1). The problem examined in the present work extends the analysis of Danilovskaya [27] to a microstructured material modeled by gradient thermoelasticity. The goal of the present study is to reveal the influence of the microstructure on the macroscopical behavior of complex materials under thermal shock conditions. It is worth noting that due to the complexity of the equations of gradient thermoelasticity very few solutions to benchmark initial-boundary problems, such as the present one, exist in the literature.

The paper is organized as follows. The equations governing the thermoelastic response of a gradient elastic solid, as derived in [23], are briefly introduced. After selecting appropriate nondimensional quantities, the respective Initial-Boundary Value Problem (IBVP) is stated. The variational form of the problem is defined and stability estimates for the weak solution are provided. Both cases of weak and strong thermoelastic coupling are analyzed. The case of a classical thermoelastic half-space may be retrieved by setting the microstructural parameters of the enhanced model to zero. In the framework of classical elasticity, the problem under consideration has been treated by many authors [27–31], and these results will be used as reference solutions for comparison between the two theories.

For the solution of the enhanced thermoelastic model, special finite elements are introduced, based on the weak formulation of the IBVP. These 3 node elements feature Hermite polynomials of 5th degree for the approximation of the displacement field. The higher regularity finite element space introduced is needed due to the higher order spatial derivatives acting on the displacement field in the governing partial differential equations. Several numerical results are presented and the convergence characteristics of the proposed numerical scheme are studied in detail. An energy balance equation is formulated based on the variational form of the weakly coupled system and the compliance of the numerical solutions with this constraint is analyzed. Moreover, the dispersive nature of the thermoelastic pulses, as dictated by the gradient elasticity theory [32–35], has been verified and found to occur in normal or anomalous type, depending on the relative magnitude of the microstructural parameters. It is shown that the resulting kinematic fields (displacements, strains) are smoother than those predicted by the classical theory of thermoelasticity [31].

2. Governing equations

In the following, we consider a homogeneous gradient elastic half-space. The half-space is assumed to be initially at uniform temperature T_o and the free surface is suddenly subjected to convection heat transfer with a surrounding fluid medium at temperature $T_\infty > T_o$. The convective heat transfer starts at time instant $t = 0$ and constitutes the only forcing