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An improved analysis of free torsional vibration of axially loaded thin-walled beams with point-symmetric open cross-section

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ABSTRACT

The objective of the paper is to analyze the influence of bimoment induced by constant axial loads on the free motion of thin-walled beams with point-symmetric open cross-section. For various boundary conditions, a closed-form solution for natural frequencies of free harmonic vibrations was derived by using a general solution of governing differential equations of motion based on Vlasov's theory. In order to investigate the effect of the bimoment on natural frequencies, the numerical examples with symmetric Z cross-section are given. The obtained results, verified using an ANSYS finite element model, demonstrate that the influence of the bimoment is important in the assessment of torsional natural frequencies.

1. Introduction

The thin-walled beam members are widely used in aerospace, automobile and civil engineering. These structures have to resist dynamic loads such as wind, traffic and earthquake loadings, so understanding their dynamic behavior and an accurate assessment of dynamic characteristics of thin-walled beam structures are extremely important. The effects due to initial axial force on dynamic response of the thin-walled beams are of particular interest. The helicopter, turbine or propeller blades, plane and space frames, as well as girders of cable-stayed bridges could be qualified as axially loaded structures. Due to their practical importance mentioned above, the vibration analysis of thin-walled beams subjected to initial axial loading has been studied by different authors (Hashemi and Richard [1]; Jun and Xianding [2]; Kim and Kim [3]; Chen and Hsiao [4]; Machado and Cortínez [5]; Borbón and Ambrosini [6]; Vo and Lee [7]; Prokić and Lukić [8]).

Starting with the assumptions of Vlasov's theory, the free vibration of axially loaded thin-walled members with open and point symmetric cross-section (a cross-section where centroid and shear center coincide) are studied. It is assumed that constant axial loads are applied to the beam ends at points with non-zero values of warping function what results in the appearance of bimoment. According to the authors' knowledge, there is a lack of investigation in this area. In numerical examples with symmetric Z-section, it is demonstrated that the bimoment produced by static axial forces shows significant effect, so the torsional natural frequencies are increased or decreased with respect to the traditional approach of analysis with the axial force and without bimoment.

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2. Theoretical formulation

The free torsional vibrations of thin-walled beam of span L with point-symmetric open cross-sections are analyzed. A constant axial force ${}^0N = P$ is assumed to act through the centroid of the cross-section that produces a variable bimoment ${}^0M_{\omega}(z) = \lambda(z){}^0M_{\omega P}$ (${}^0M_{\omega P} = P\,\omega_{(P)}$ denotes the end bimoment, while $\omega_{(P)}$ is the value of the warping function at the loading point) where $\lambda(z)$ is a static bimoment distribution function according to the first order theory (see Appendix A).

As already mentioned in the introduction, in the case of point-symmetric cross-section the shear center and centroid coincide ($x_D = y_D = 0$) and the equations of motion (B16) from Appendix B are uncoupled, so that the equation of torsional vibration,

$$EI_{\omega\omega}\varphi^{IV} - GK\varphi'' - Pi_D^2\varphi'' - P\beta_\omega\omega_{(P)}\left(\lambda\varphi'\right)' - \rho I_{\omega\omega}\ddot{\varphi}'' + \rho A i_D^2\ddot{\varphi} = 0 \tag{1}$$

can be analyzed independently. In Eq. (1), $\varphi(z,\ t)$ is the angle of twist, E and G are Young's modulus and shear modulus respectively, ρ is the mass density, K is St. Venant's torsional stiffness, $I_{\omega\omega}$ is warping stiffness, A is the cross-sectional area, while the remaining geometric properties of the cross-section are defined in the Appendix B. The differentiation with respect to z is denoted as $(\ )'$ while $\dot{\varphi} = \partial\varphi/\partial t$ where t is time. In Eq. (1), the term $-P\beta_\omega\omega_{(P)}(\lambda\,\varphi')'$ represents the initial bimoment distribution which influence on vibrations has not been reported by other authors. Note that influence of bimoment exists only in the cross-sections without axes of symmetry, i.e. when $\beta_\omega \neq 0$. This requirement is fulfilled for the point symmetric open cross-sections excluding the sections with double symmetry.

The angle of twist function $\varphi(z,\ t)$ of the thin-walled beam vibrating under an axial load and a bimoment can be expressed as a superposition of two independent functions,

$$\varphi(z,t) = \varphi_0(z) + \varphi_1(z,t). \tag{2}$$

The twist angle $\varphi_0(z)$ represents fundamental – static state of the beam, and it is defined by equilibrium equations according to the linearized second order theory [9],

$$EI_{\omega\omega}\varphi_0^{IV} - GK\varphi_0'' - Pi_D^2\varphi_0'' - P\beta_\omega\omega_{(P)}\left(\lambda{\varphi'}_0\right)' = 0 \tag{3}$$

and appropriate boundary conditions at $z = 0$ and $z = L$. The function $\varphi_1(z,\ t)$ is the twist angle of dynamic state due to vibration about the fundamental state and it satisfies differential equation of motion (Eq. (1)),

$$EI_{\omega\omega}{\varphi_1}^{IV} - (GK + Pi_D^2 + P\beta_\omega\omega_{(P)}\lambda_m)\varphi_1'' - \rho I_{\omega\omega}\ddot{\varphi}_1'' + \rho A i_D^2\ddot{\varphi}_1 = 0 \tag{4}$$

with homogeneous boundary conditions. In order to obtain a closed form solution and engineering assessment for natural frequencies, in Eq. (4) a constant value λ_m (the mean value of $\lambda(z)$ along the beam axis) is assumed for $\lambda(z)$. The applicability of this assumption will be tested in numerical examples for various boundary conditions.

For a free vibration analysis, a sinusoidal variation of φ_1 with circular frequency p is assumed,

$$\varphi_1(z,t) = \Phi(z)\sin pt, \tag{5}$$

where $\Phi(z)$ is the amplitude of sinusoidal varying twist. Substituting Eq. (5) into Eq. (4) gives

$$EI_{\omega\omega}\Phi^{IV} - \left[GK + P\left(i_D^2 + \beta_\omega\omega_{(P)}\lambda_m\right) - \rho p^2 I_{\omega\omega}\right]\Phi'' - \rho A i_D^2 p^2\Phi = 0. \tag{6}$$

The differential Eq. (6) can be written as:

$$\Phi^{IV} - H_1\Phi'' - H_2\Phi = 0, \tag{7}$$

where

$$H_1 = \left(\frac{GK}{EI_{\omega\omega}} + \frac{P\left(i_D^2 + \beta_\omega\omega_{(P)}\lambda_m\right)}{EI_{\omega\omega}} - \frac{\rho p^2}{E}\right)$$

$$H_2 = \frac{\rho A i_D^2 p^2}{EI_{\omega\omega}}. \tag{8}$$

The general solution of linear ordinary differential Eq. (7) has two pairs of conjugate complex roots. The solution has the following form:

$$\Phi(z) = C_1\sinh\alpha z + C_2\cosh\alpha z + C_3\sin\beta z + C_4\cos\beta z, \tag{9}$$

where C_i $(i = 1–4)$ are undetermined constants, while coefficients α and β are

$$\alpha = \pm\sqrt{\frac{1}{2}H_1^2 + \frac{1}{2}\sqrt{H_1^2 + 4H_2}}$$

$$\beta = \pm\sqrt{\frac{1}{2}\sqrt{H_1^2 + 4H_2} - \frac{1}{2}H_1^2}. \tag{10}$$