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Bifurcation and chaos analysis for multi-freedom gear-bearing system with time-varying stiffness

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ABSTRACT

This study performs a systematic dynamic analysis of a gear-bearing system with time varying stiffness, gear backlash and surface friction. At first the period expansion method is proposed to build a six-degree-of freedom nonlinear dynamic model of a spur gear pair. Then the dynamic orbits of the system are observed using the bifurcation diagrams with the mesh stiffness and the rotational speed ratio as control parameters. And the onset of chaotic motion is identified from the phase diagrams, FFT spectra, Poincaré maps and the largest Lyapunov exponents of the gear-bearing system. The numerical results reveal that the system enters into chaotic motion under several frequency jumps with the increase of excitation frequency. When the support stiffness is raised, the number of frequency jumps increases, and the system exhibits a diverse range of periodic, sub-harmonic, and chaotic behaviors. In this study the results provide a useful source of reference for engineers and technicians in designing and controlling such systems.

1. Introduction

Gear mechanisms have extensive application in modern power transmission system, due to their considerable technical advantages. Gear pair dynamics affects the noise, vibration and durability performances of gearboxes and power transmission system. Owing to the high sensitivity of the gear pair to time-varying stiffness, tooth profile errors, and overall structural dynamics, the vibrational response of the gear pair system can be very complex and not easy to control. Therefore, it is essential to gain a more thorough understanding of the gear pair dynamics in the design and control of the transmission system. In this paper, we focus on the multi-freedom model and systematic dynamic analysis of a gear-bearing system with time varying stiffness, gear backlash and surface friction.

In order to develop the gear transmission with low noise and high performance, many studies have focused on analyzing gear dynamics or relative areas of gear transmission system. Ozguven and Houser [1] provided a mathematical model for simulating the dynamics of gear transmission system by using static error. Based on this research, Ozguven [2] developed a 6 degrees of freedom (DOF) model for a spur gear pair and performed a dynamic analysis on gears considering the effects of damping, backlash, single and double-side impacts, and tooth profile modification. Kahraman et al. [3-4] presented a series of studies in the influence of several factors on the responses of the nonlinear gear system, such as bearing clearances, time-varying mesh stiffness and external excitation, and these analyses are based on 1DOF or 3DOF model. Wang et al. [5] carried out the dynamic analysis of a hypoid gear pair considering the effects of time-varying mesh parameters and backlash. Amabili and Rivola [6] applied a time-varying meshing damping method to analyze the dynamic response of a spur gear pair

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Nomenclature

α	the pressure angle
t	meshing time
k_m	averaged meshing stiffness
k_h	nonlinear meshing stiffness
c_h	the damping coefficient
c_1, c_2	damping of bearing
b_h	backlash
r_{ai},r_{bi}	base circle radius
ω_1, ω_2	angular velocity
F_{meshi} $(i = 1, 2)$	dynamic meshing force
$f(x)$	the function of non-linear force displacement
f	friction force
DMF	dynamic meshing force
$\delta(t)$	dynamic transmission error (DTE)
μ_i	the friction coefficient of the tooth surface
λ_i	the direction coefficient of the friction force
$S_{1i}(t)$,$S_{2i}(t)$ $(i = 1, 2)$	the function of friction moments
m_1, m_2,	gear masses
I_1, I_2	gear inertias
T_1, T_2	input torque and output torque
ε	the contact ratio of the gear pairs
$\xi_1(t)$,$\xi_2(t)$	the periodic functions of friction moments
$\rho_1(t)$,$\rho_2(t)$	the periodic functions of the direction coefficient of the friction force
x_i, y_i, θ_i	the bearing displacement and the dynamic angular displacement of the gears

system. Velex [7] presented an alternative formulation for the definition of profile modifications in high-contact-ratio spur gears to select optimum relief with regard to transmission error (TE) fluctuations over a range of loads. Theodossiades and co-workers [8] used an analytical method to characterize damping and stiffness in lightly loaded, lubricated gear pairs at different operating speeds and lubricant temperatures. Zhao and Ji [9] studied the torsional vibrations of wind turbine gearbox having two planetary gear stages and one parallel gear stage. In 2000, Parker et al. [10] applied a unique finite element-contact method to analyze the nonlinear dynamics of a spur gear system and verified the numerical results experimentally. Gill-Jeong [11] studied the nonlinear behavior of spur gear pairs with one-way clutch and concluded that the clutch could soften non-linearity and jump phenomena. Bonori et al. [12] developed an original application of Genetic Algorithms (GAs) to optimize spur gear pairs toward vibration and noise reduction. He et al. [13] developed a spur gear pair model with sliding friction and rectangular mesh stiffness, and analyzed the relationship between sliding friction and mesh stiffness by using the multi-term harmonic balance methods. Shen et al. [14] utilized an incremental harmonic balance method to build a single degree-of-freedom dynamic model for a spur gear pair with time-varying stiffness and a static transmission error. Chang-Jian and Hsu [15] performed a systematic analysis of the dynamic behavior of a gear-bearing system with the turbulent flow effect, nonlinear suspension, nonlinear oil-film force and nonlinear gear mesh force. Guo and Parker [16] investigated the dynamics of planetary gears where nonlinearity is induced by bearing clearance. Walha et al. [17] investigated the dynamics of a two-stage gear system involving backlash and time-dependent mesh stiffness, which were different from many previous single-stage models. The influences of damping coefficient, excitation frequency, and backlash on bifurcation and chaos properties of the multistage planetary system are analyzed by Li et al. [18]. He and Singh [19] theoretically and experimentally demonstrated that the friction forces also played an indispensable role in the dynamic behavior of the gear-bearing system. Considering backlash, transmission error, time-varying mesh stiffness, and layout parameters, Gao and Zhang [20] studied the nonlinear vibration characteristics of geared rotor bearing system and the interactions among gears, shafts, and plain journal bearings. Recently, Chang-Jian [21] presented studies of nonlinear analysis for gear-bearing system with nonlinear suspension, which provided a detailed understanding of the nonlinear dynamic response for the gear-bearing system.

Though the previous works emphasized and verified the significance of gear dynamics, litter previous research was found on the nonlinear dynamic analysis of spur gear pair systems coupling with bearing system. Although some nonlinear factors of the gear system such as the bearing clearance or the oil film force were considered in dynamic analysis, less research has been carried out in the effect of support stiffness on the bifurcation characteristic and chaotic motions of the spur gear transmission system with the time varying mesh stiffness, friction force and backlash. Many studies actually simplified non-linear physical phenomena to a linear form given a sufficiently precise linearization technique. However, this simplification is improper for high rotational speed, high-power gear systems, and the application in the design, and analysis stage may lead to a flawed or potentially dangerous operation. Accordingly, nonlinear analysis methods are commonly preferred within

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