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# Vertical vibration of a rigid strip footing on a transversely isotropic multilayered half-plane

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ABSTRACT

A rigorous analytical method is developed to analyze the vertical vibration of a rigid strip footing on a transversely isotropic multilayered half-plane. Based on mixed boundary conditions for the interaction problem and the analytical layer-element solution for the multilayered half-plane, a pair of dual integral equations of contact stress is derived in the Fourier transform domain. By means of Jacobi orthogonal polynomials, the dual integral equations are converted to a system of linear equations and further solved. Fourier inverse transform is carried out to obtain the final results in the frequency domain. Comparisons with the existing solutions confirm the accuracy of the proposed method and more examples are given to illustrate the influence of material anisotropy and stratification.



## 1. Introduction

The contact problem between a rigid plate and an elastic isotropic or orthotropic medium plays an important role in the study of soil-structure interaction. As a classical example of soil-structure interaction problems in civil engineering, the static analysis of a rigid plate and an elastic medium is useful for understanding the behavior of foundations subjected to external loads. Harding and Sneddon [1] tackled the problem of the frictionless normal indentation in a half-space by a rigid punch as early as 1945. Collins [2] considered the interaction of a thin rigid inclusion embedded in an isotropic elastic solid under prescribed displacements. More follow-up works could be found in works of Keer [3], Selvadurai [4], Pak and Gobert [5].

On the other hand, its dynamic analog also has important practical values, which provides theoretical reference and support for the study of seismology, earthquake engineering, machine vibrations, and dynamic hardness testing. Robertson [6] extended the work of Harding and Sneddon [1] to analyze the dynamic problem. The vertical dynamic response of a rigid disc rest;ing on an elastic half-space was also studied by Bycroft [7] and Lysmer [8]. Later, Lysmer and Kuhlemeyer [9] investigated the displacement function of a rigid circular disc resting on or partially embedded in an elastic half-space. The steady motion of a rigid strip bonded to an elastic half-space can be found in Oien [10]. Luco and Westmann [11,12] analyzed the dynamic response of a rigid circular footing and a rigid strip footing bonded to a half-space, respectively. Later, Luco and Mita [13] presented the impedance functions for axial, torsional, horizontal, and rocking motions of a circle foundation. Hryniewicz [14] also studied the dynamic response of a rigid strip on an elastic half-space, and further presented

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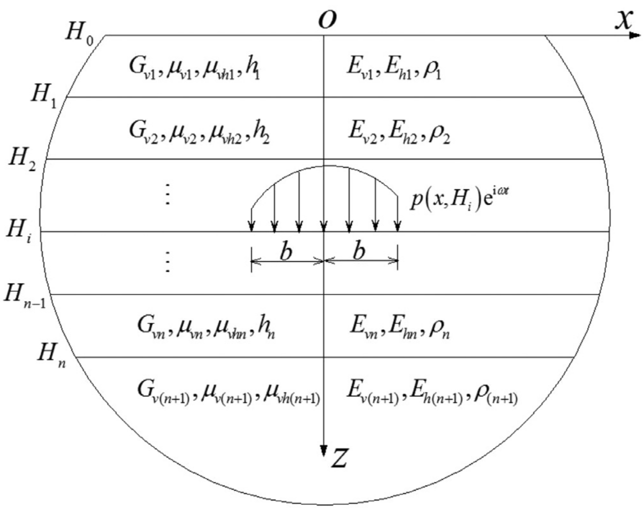

**Fig. 1.** A transversely isotropic multilayered half-plane.

the contact stress distribution at the interface. Pak and Gobert [15] considered the vertical vibration of a rigid disc buried at a depth below the surface of an isotropic half-space.

The papers mentioned above are all focused on the response of the elastic half-space. In order to simulate the real situation, the effect of the soils' anisotropy should be taken into consideration. Soils generally take on the phenomenon of transversely isotropy and layering due to long-term sedimentation processes. Many researches, as in Payton [16], Rajapakse and Wang [17,18], Yang et al. [19], Eskandri-Ghadi [20], Rahimian et al. [21] and Khojasteh et al. [22] provided fundamental solutions for a transversely isotropic half-space. Solutions for a transversely isotropic multilayered medium under dynamic loads can be found in Khojasteh et al. [23,24], Ai et al. [25], Ai and Li [26], Ai and Zhang [27]. Therefore, these fundamental solutions can act as the basis of interaction problems between a rigid plate with a transversely isotropic half-space or multilayered medium.

Kirkner [28] developed an analytical solution for the forced vibration of a rigid disc on a constrained transversely isotropic elastic half-space. With the aid of the Green's functions presented by Eskandri-Ghadi [20], Eskandri-Ghadi et al. [29,30] investigated the vertical vibration of a rigid circular disc attached to the surface and buried in an arbitrary depth of a transversely isotropic half-space. Later, Eskandri-Ghadi et al. [31] extended their solutions into a multilayered half-space. Recently, Ai and Liu [32] studied the axisymmetric vibration of an elastic circular plate bonded on a transversely isotropic multilayered half-space.

In practical engineering, strip foundations and dams are generally considered as the plane strain problems, so the dynamic response of a rigid strip footing should receive attention. Gazatas [33] presented a semi-analytical formulation to study the dynamic response of a rigid strip footing supported on the surface of layered cross-anisotropic soils. Lin et al. [34] proposed the precise integration method developed by Zhong et al. [35], for the dynamic stiffness matrices of a rigid strip footing resting on an arbitrary anisotropic layered stratum. The solutions of vertical vibration of a rigid strip footing on a transversely isotropic multilayered half-plane up to now are semi-analytical or numerical ones, so the purpose of this paper is to provide mathematically rigorous analytical results for this problem by the dual integral equation method, which will offer a better understanding of the essence of the problem. Based on the mixed boundary conditions for the interaction problem and the analytical layer-element solution for a multilayered half-plane [27], a pair of dual integral equations of contact stress is derived. Similar to the method in Ai and Zhang [36] for a static case, the dual integral equations are solved by virtue of the Jacobi orthogonal polynomials. Selected numerical results are performed to demonstrate the accuracy of present method, and to discuss the influence of material anisotropy and stratification in different frequencies of excitation.

## 2. The analytical layer-element solution for a multilayered half-plane

In a Cartesian coordinate system, defined that the $z$-axis is normal to the plane of isotropy, an $n$-layered transversely isotropic elastic soil system with an underlying half-plane is illustrated in Fig. 1. The thickness of the $i$thlayer is $h_i = H_i - H_{i-1}$, where $H_i$ and $H_{i-1}$ are the depths from the surface to the bottom and top of the $i$th layer, respectively. $E_{vi}$, $E_{hi}$ and $G_{vi}$ are the vertical Young's modulus, horizontal Young's modulus and shear modulus of the $i$th layer, respectively. $\mu_{vhi}$ and $\mu_{hi}$ are Poisson's ratios characterizing horizontal strain due to the stress acting parallelly and normally to the plane, respectively. In addition, $\rho_i$ denotes the density of the $i$th layer. An arbitrary time-harmonic distributed load $p(x, H_i)e^{i\omega t}$ of width $2b$ is applied at the depth of $H_i$, where $\omega$ is the circular frequency and $t$ is the time variable.