Applied Mathematical Modelling 000 (2016) 1–15

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Explicit quasi-optimal inspection schemes from nonconformity count data with controlled producer and consumer risks

Arturo J. Fernández*

Departamento de Matemáticas, Estadística e Investigación Operativa, Universidad de La Laguna, La Laguna 38071, Canary Islands, Spain

ARTICLE INFO

Article history:
Received 24 January 2014
Revised 19 June 2016
Accepted 3 August 2016
Available online xxx

Keywords:
Poisson model
Taylor series expansion
Constrained optimization
Average producer and consumer risks
Quality control

ABSTRACT

Assuming that maximum allowable risks are specified for both producer and consumer, an integer nonlinear programming problem is formulated in order to find the optimal inspection scheme for lot acceptance purpose based on defect count data and limited prior information. The number of nonconformities per unit is assumed to follow a Poisson distribution. Closed-form quasi-optimal inspection schemes for screening submitted lots of manufactured material are determined using normal transformations of chi-square distributions and second-order Taylor series expansions of the operating characteristic function. The full prior model on the process average is not needed. Instead only prior estimations of the means and variances for the quality levels of the acceptable and rejectable lots are necessary to derive explicit and accurate approximations of the smallest number of units to be tested per lot and the maximum tolerable number of nonconformities in the selected sample. The suggested methodology requires little prior information to judge the quality of the submitted material and greatly simplifies and quickens the determination of optimal acceptance sampling plans with controlled average risks. In general, the inclusion of prior knowledge yields substantial savings in sample size, as well as improved evaluations of the actual average producer and consumer risks. Moreover, our approach is quite robust against slight changes in the available prior knowledge and also provides a suitable way to properly combine multiple expert judgments. The developed procedure is applied to the manufacturing of paper and glass to show the accuracy of the proposed quasi-optimal plans and the corresponding average risks.

1. Introduction

The planning of inspection schemes for acceptance purposes is a typical problem arising in industrial quality control. Acceptance sampling plans are customarily developed to determine decision rules in order to accept or reject production processes or submitted lots based on random sample data. Available inspection schemes in the literature apply diverse decision criteria. Some recent papers are Baklizi and El Masri [1], Pearn and Wu [2], Hong et al. [3], Arizono et al. [4], Fernández [5–8], Lu and Tsai [9], Pérez-González and Fernández [10], Seo et al. [11], Fernández et al. [12], Elsayed [13],

* Fax: +34 922 318170.
E-mail address: ajfernan@ull.es

http://dx.doi.org/10.1016/j.apm.2016.08.005

Please cite this article as: A.J. Fernández, Explicit quasi-optimal inspection schemes from nonconformity count data with controlled producer and consumer risks, Applied Mathematical Modelling (2016), http://dx.doi.org/10.1016/j.apm.2016.08.005

Fernández and Pérez-González [14,15], Guo and Liao [16], Wu et al. [17–19], Aslam et al. [20–22], Yang [23], Liu et al. [24], Wu and Liu [25] and Lee et al. [26].

The design of the optimal acceptance test plan usually assumes an agreement between the producer and the consumer on the specification of the maximum tolerated risks. In general terms, producer risk is the probability of rejecting good quality material and consumer risk is the probability of accepting poor quality material. Essentially, the determination of the best inspection scheme can be formulated and solved as a constrained optimization problem.

Quality evaluations are routinely carried out by using discrete attributes or continuous variables. In practice, acceptance sampling by attributes is the most widely applied inspection procedure. In many cases, each sampled unit is simply classified as defective or nondefective. In some situations, the inspected items may present more than one defect. Acceptance test plans for defects per unit commonly depend on the total number of nonconformities observed in the randomly selected sample. The terms nonconformance or nonconformity are often employed to make clear that failure to meet a requirement does not automatically indicate that the examined unit is unsuitable for use.

The Poisson model is a discrete probability distribution usually adopted in practical applications to describe the number of nonconformances in items of specific size or the number of failures in a fixed period of time [27–34]. The number of nonconforming units manufactured in a given length of time, the number of blemishes in sheets of glass of fixed size, the number of imperfections per unit of area in inspecting cloth, and the number of flaws in painted sheets of metal could be modeled by Poisson distributions.

In practice it is generally advantageous to properly combine available sample data with existing technical information and experience [35–47]. Nonetheless, a fundamental supposition in conventional inspection from Poisson count data is that the process average λ is a constant number. In most practical cases, the assumption of a fixed defect rate λ cannot be justified, even when the production process is in control. Previous evidence and expert judgment on the process average can be of great practical value in designing optimal acceptance plans for defects per unit. In many circumstances, however, the available prior information is not sufficient to completely determine an appropriate prior distribution on λ.

Recently, Fernández [48] studied the design of inspection schemes for defects per unit with fixed acceptance numbers and limiting quality levels, including an introductory survey on acceptance sampling by continuous variables and discrete attributes. The optimal plan proposed in [48] provides the appropriate protection to the customer without considering a controlled producer risk. The present paper deals with the design of the optimal acceptance sampling plan based on nonconformity count data and minimal prior information on the quality level which keeps the average producer and consumer risks below predetermined thresholds. The number of nonconformities per sampled unit is assumed to follow a Poisson model with unknown parameter λ, which represents the average number of defects per unit. All nonconformances are inherently considered to have virtually indistinguishable effects on the finished product. The inclusion of prior knowledge into the decision analysis provides substantial savings in sample size, as well as more reliable evaluations of the actual producer and consumer risks.

A general procedure for obtaining a closed-form approximation of the optimal inspection scheme is presented, and its accuracy is discussed. The method is based on normal approximations of chi-square distributions and second-order Taylor series expansions of the operating characteristic (OC) function around the expected quality levels of the acceptable and rejectable lots. Our results notably simplify the determination of optimal plans from nonconformity count data and prior information. As the proposed design is nearly optimal, it can serve as a starting point in iterative search procedures. Moreover, it may be used to explain the influence of the acceptable and rejectable quality levels, the maximum tolerable risks and the prior knowledge on the optimal inspection scheme. Furthermore, our procedure is reasonably insensitive to small disturbances in the prior information. In general, the explicit approximation is very easy to implement and only requires information on the prior means and variances of the quality levels of the acceptable and rejectable lots. Since this information could be estimated from past data and expert opinions, there is no need to specify the full prior model on λ to find a quasi-optimal acceptance sampling plan.

The remainder of this paper is organized as follows. Section 2 outlines the classical inspection schemes for nonconformities per unit and their probabilistic models. The average producer and consumer risks, as well as the corresponding OC functions, are defined in Section 3. An integer nonlinear programming problem is formulated in Section 4 to find the optimal inspection scheme with controlled average risks. Next, Section 5 presents a simplified procedure to determine nearly optimal acceptance test plans based on minimal prior information. Plausible approximations to the best inspection scheme are derived in closed-forms. For illustrative purposes, the methodology is applied in Sections 6 and 7 to the manufacturing of glass and paper, respectively, to show the accuracy of the proposed quasi-optimal plans and the corresponding average risks. Section 8 then offers some conclusions, a brief discussion and several suggestions for future research.

2. Inspection schemes for nonconformity count data

Assume that in some manufacturing process the number of defects X in a product follows a Poisson distribution with mean λ. Consider that N units are randomly selected from a large lot of products, and also that X_i represents the number of defects which are observed in the ith unit for $i = 1, \ldots, N$. The likelihood function of the defect rate λ is then proportional to $\exp(-N\lambda)\lambda^Y$, where $Y = X_1 + \cdots + X_N$ denotes the total number of defects in the sample. Clearly, all the sample information on the process average λ is contained in the minimal sufficient statistic Y.