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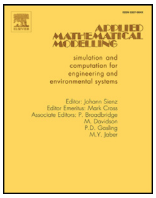

# Analytical modelling of the bending behaviour of hybrid composite-concrete beams: Methodology and experimental validation

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ABSTRACT

The advantages of high performance composite materials have stimulated their use as reinforcement in concrete structures as well as the research on hybrid composite-concrete elements. For the design of these hybrid structures – with often complex geometries – the standard concrete calculation rules, based on official codes such as Eurocode 2, are often not elaborated enough. Moreover, these standard models are limited or simplified to a linear behaviour and do not even allow the integration of nonlinear reinforcement behaviour. In this paper, the authors propose a general calculation methodology to simulate analytically the structural behaviour of hybrid beams, including their ultimate loadbearing capacity and deflection. This methodology introduces an original search algorithm that reduces the computation time significantly without losing accuracy. The analytical model is validated by ten experiments on 3-m-span hybrid beams with differing geometry. The experiments demonstrate the accurate prediction of the analytical model both for the load–deflection behaviour and for the occurring strains, and confirm the model assumptions made. Moreover, the structural feasibility of the proposed hybrid beams is demonstrated as they exhibit a sufficient loadbearing capacity and stiffness. This structural performance in combination with the ability to predict the behaviour reveals a great potential for future structural applications.

## 1. Introduction

Reinforced concrete (RC) is one of the most popular building materials nowadays. This results in the availability of detailed design formulas, such as Eurocode 2 [1] for RC structures to calculate for example the loadbearing capacity in Ultimate Limit State (ULS) or the deflection in Serviceability Limit State (SLS). The elaboration of these design formulas, based on commonly used cross-sectional shapes (rectangular, T-shape) and materials (concrete and steel) has led to generic equations, depending on the general geometry and material parameters.

Thanks to their tailorable characteristics, low self-weight and good protection against environmental corrosion, the use of high performance composite materials has increased over the last decades. Within the construction industry, composites are mostly used as external reinforcement [2,3]. This increased interest in composites has also stimulated the research on

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## Nomenclature

| | |
|---|---|
| $b$ | index for ‘box’ |
| $c$ | index for ‘compression’ |
| ca | index for ‘carbon’ |
| $co$ | index for ‘concrete’ |
| $f$ | index for ‘fibre’ |
| $ipc$ | index for ‘GFR.IPC’ |
| $lb$ | index for ‘lower box’ |
| $m$ | index for ‘mean’ |
| $t$ | index for ‘tension’ |
| $ub$ | index for ‘upper box’ |
| $a$ | distance between support and point load |
| $b'_{i,mat}$ | virtual width of the discretised height element i for the considered material |
| $b_{i,mat}$ | width of the discretised height element i for the considered material |
| $E'_f$ | effective stiffness of fibres |
| $E_{co}$ | concrete stiffness, based on secant modulus |
| $E_f$ | stiffness of fibres |
| $EI$ | bending stiffness of a beam’s cross-section |
| $E_{ipc,bc}$ | stiffness of the GFR.IPC in tension before cracking and in compression |
| $E_{ipc,pc}$ | stiffness of the GFR.IPC in tension after cracking |
| $E_m$ | stiffness of the IPC matrix |
| $E_{ref}$ | reference stiffness to create a virtual cross-section ($=E_{co}$) |
| $f$ | maximum stress |
| $F$ | force |
| $f_{co,c,m}$ | experimental mean strength of concrete |
| $h$ | height |
| $L$ | length of the beam |
| MOI | moment of inertia (second area moment) |
| $q$ | line load due to self-weight of the beams |
| $t$ | thickness |
| $v_f$ | fibre volume fraction |
| $v_m$ | matrix volume fraction |
| $w$ | width |
| $\Delta_{strain}$ | step size to increase the strain |
| $\varepsilon$ | strain |
| $\bar{\varepsilon}$ | maximum strain |
| $\varepsilon_{bottom}$ | strain at the bottom most fibre of the cross-section |
| $\varepsilon_{co,c,p}$ | concrete strain at the end of the parabola/begin of rectangle |
| $\varepsilon_{ipc,cr}$ | strain in the GFR.IPC at the moment of cracking |
| $\varepsilon_{top}$ | strain at the top most fibre of the cross-section |
| $\sigma$ | stress |
| $\sigma_{ipc,cr}$ | stress in the GFR.IPC at the moment of cracking |

structural hybrid elements, which are composed of both concrete and composites. This intensive research has led to several hybrid beam designs [4,5] that combine the low cost of concrete with the good structural performances of composites. Moreover, the low self-weight of these composites can facilitate the installation and consequently decrease the installation time and thus cost [6]. These hybrid designs, combining the benefits of different materials have not remained limited to beam elements, but have been extended to full floors [7,8].

Multiple reasons complicate the determination of generic design equations for hybrid composite-concrete elements: they are composed of (i) different materials and (ii) shaped in a complex geometry, according to their specific material characteristics, contributing in the most efficient way to the internal force distribution along the structural element [9]. Moreover, (iii) to stimulate the industrial uptake of hybrid elements, optimised shapes and dimensions are being designed. Optimising and thus iteratively varying the material use and the element shape requires a general calculation method instead of specific ‘case-dependent’ design formulas.

An increased interest in the calculation of hybrid beams is proven by previous research. Promis et al. [10] proposed a design methodology to calculate the stresses and deflections in composite beams. Due to iterative stiffness calculations (and thus a high calculation time), Ferrier et al. [11] used an analytical model that omits these iterations by scanning the full strain field. For nonlinear material properties, this iterative strain scanning method becomes however labour intensive. This

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