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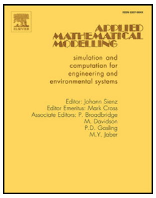

Analytical solution of the induced currents in multilayer cylindrical conductors under external electromagnetic sources

Jesús Acero[a,*], Claudio Carretero[b], Ignacio Lope[c], Rafael Alonso[b], José Miguel Burdío[a]

[a] Departamento de Ingeniera Electrónica y Comunicaciones, Universidad de Zaragoza, Zaragoza 50018, Spain
[b] Departamento de Física Aplicada, Universidad de Zaragoza, Zaragoza 50009, Spain
[c] B/S/H/ Home Appliances, Zaragoza 50016, Spain

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ABSTRACT

We present a closed-form solution for the induced losses in round conductors consisting of several concentric layers. The geometry under study corresponds to an infinitely-long and isolated multilayer cylinder where layers can have different electromagnetic properties and the number of layers is not restricted. The multilayer conductor is under an external time-varying magnetic field which induces currents and, accordingly, generates Joule dissipation. Total induced losses are obtained by integrating the losses of each layer. Mathematical expressions of the current distribution in each layer are derived from the solution of Maxwell's equations. These expressions consist of a combination of Bessel functions of different kinds and orders. The current distribution in a particular layer not only depends on the properties of the layer but also on the properties of the rest of layers. Consequently, matrix formalism is adopted for describing current distribution of layers. Matrix description is numerically solved and results are compared with finite element simulations for different arrangements and cases.

1. Introduction

Tubular conductors are used in many applications at medium-high frequencies, ranging from several kHz up to tens of MHz, in order to alleviate the ac losses, caused by skin and proximity effects. Some popular applications of tubular conductors are induction furnaces or magnetic elements for broadcasting or radio-frequency systems. Apart from the loss reduction, tubular conductors also possess convenient mechanical and thermal properties. Considering induction furnaces, inductors made of copper tubes can adapt to the shape of the work-piece without including bobbins or holders. Moreover, hollow tubes can be cooled by circulating a coolant element, and the operation temperature of bare copper conductors is higher than the temperature limit of insulated magnet wires. These properties make copper tubular conductors potentially attractive for other closely-related applications in the mid-power range, such as wireless power transfer (WPT) systems.

Modeling ac losses in tubular conductors has been addressed by several authors. Dwight proposed several ac loss models for tubes and other shapes mainly based on the solution of the current density in the conductor. In general, his works described approximate solutions mainly based on asymptotic tendencies at both low and high frequencies [1–5].

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[*] Corresponding author.
E-mail address: jacero@unizar.es (J. Acero).

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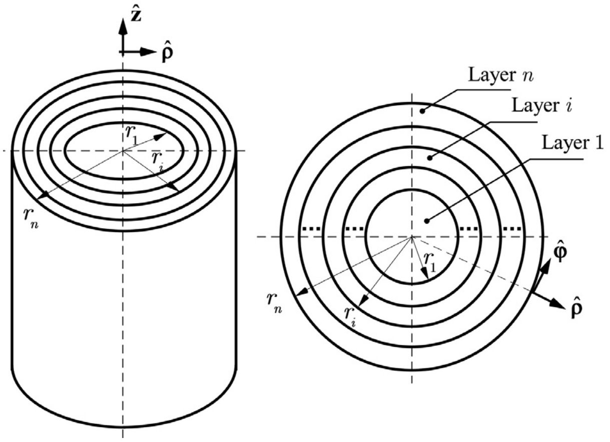

Fig. 1. Geometry of a multilayer cylindrical conductor.

Similarly, Arnold developed an approximate solution based on Dwight's results [6]. This work was mainly focused on the study of copper-clad or aluminum-clad (bimetallic) conductors. Schelkunoff studied the case of the coaxial transmission line and developed an exact formula for the internal impedance of a tubular conductor in terms of modified Bessel functions [7]. Teare and Webb proposed a model of the skin effect of solid conductors consisting of several materials, in this case a bimetallic arrangement [8]. This study addressed the case of hollow conductors but an analysis of the loss induced by an external magnetic field was not provided. The diffusion of fields in concentric rods of different properties was also studied in [9]. Clogston introduced the idea of using multilayer structures in order to reduce the skin effect and proposed a theory for laminated planar transmission lines [10]. Other authors extended this concept to coaxial lines with the purpose of loss reduction [11]. The current density of cylindrical shells due to axial currents was obtained by solving Maxwell's equations through the magnetic vector potential and boundary conditions [12]. The methodology followed in this reference has been applied in the study presented in this paper.

More recently, some authors have proposed numerical solutions for the ac losses in tubular conductors mainly oriented to overcoming the convergence problems of Bessel functions for large arguments [13–17]. Moreover, cylindrical conductors arranged with multiple concentric layers have been proposed to reduce the skin effect of litz wires [18] and the skin effect in inhomogeneous conductors has been also studied [19]. Therefore, interest in this kind of conductors is currently observed.

Despite the substantial number of works on electromagnetic fields involving tubular conductors, it seems that there is still more to contribute in relation to multilayer round conductors, which is the main objective of this paper. This study could be also useful for predicting the losses due to the so called skin and proximity effects.

The remainder of the paper is organized as follows. The conditions assumed in this analysis are presented in Section 2. In Section 3 the impedance of a multilayer round conductor under a longitudinal alternating voltage is derived. Section 4 is focused on obtaining fields and losses in a multilayer round conductor in a transverse varying magnetic field. Section 5 describes the validation of the analytic model by means of FEA simulations for different cases. Finally, Section 6 summarizes the contributions of the present work.

2. Conditions of the study and fundamental field equations

The analysis of power induced currents in multilayer round conductors is carried out adopting the following assumptions. First, a single conductor is considered in order to simplify the analysis. The conductor is an infinitely-long isolated cylinder and curvature in the longitudinal direction is not considered. Second, the conductor consists of concentric layers of a linear, homogeneous and isotropic material. The properties of the ith layer are σ_i, μ_i, ε_i and its external radius is r_i, as shown in Fig. 1. Third, an external harmonic ($e^{j\omega t}$ time dependence) electromagnetic field is applied and the effect of the field in the conductor and the sources of the field are decoupled. Fourth, the magneto-quasistatic approach (MQS) is assumed in this analysis. At the frequency range of interest of the mentioned applications (from dc to several MHz) the diffusion of the fields inside the conductor dominates the radiation, or equivalently, the displacement current in the system can be neglected compared to the induced currents in the conductor [20]. Considering the preceding assumptions, the electromagnetic field of the system can be described by the following magnetic vector potential equation:

$$\nabla^2\mathbf{A} + k^2\mathbf{A} = 0, \tag{1}$$

where k is the wave number and $\mathbf{A}$ is the magnetic vector potential. The wave number is defined as:

$$k = \sqrt{-j\omega\mu\sigma} \tag{2}$$

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