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Controlling posterior producer and consumer risks in lot reinspection

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A B S T R A C T

Assuming that maximum tolerable posterior risks are specified for both producer and consumer, an integer nonlinear programming problem is formulated and solved in order to determine the optimal defects-per-unit acceptance sampling plan when lots found unacceptable may be resubmitted for reinspection. The number of nonconformities per inspected item follows a Poisson distribution. A computational algorithm is proposed to solve the underlying constrained minimization problem. The suggested procedure simplifies and quickens the determination of the inspection scheme for resubmitted lot acceptance with limited posterior risks that minimizes the expected number of sampled items per lot. An application to the manufacturing of paper is considered to illustrate the methodology developed. The generalized truncated gamma distribution is used to describe the prior uncertainty about the incoming defect rate per unit. The degree of similarity between the available previous information and the current study is also evaluated. Suitable ways are provided to assume a reduced parameter space for the defect rate and to update the prior model using past performance of the acceptance plan. The incorporation of lot resubmissions, as well as previous defect count data and expert opinions, into the decision process often yields appreciable savings in sampling effort.

1. Introduction

Sampling inspection is extensively applied in industry to judge the acceptability of lots of incoming materials and outgoing products. The planning of the best inspection scheme for lot acceptance purposes can basically be viewed as a constrained optimization problem. Many acceptance sampling plans are available in the scientific literature for a wide variety of situations. Some recent papers are Baklizi and El Masri [1], Chen et al. [2], Balamurali and Jun [3], Arizono et al. [4], Tsai et al. [5], Lee et al. [6], Lu and Tsai [7], Fernández [8], Fernández and Pérez-González [9,10], Aslam et al. [11], Hsieh and Lu [12], Wu and Liu [13] and Wu and Huang [14].

In various contexts, the inspected units may present more than one defect or nonconformity, but the existence of some imperfections in an item does not instantly imply that it is unsuited for use. The stochastic behavior of the number of defects detected in a unit is often described by the Poisson distribution; see Fernández [15] and references therein. For example, the Poisson distribution is usually employed to model the number of nonconformities per unit of specific size in inspecting glass, steel, paper, plastics, cloth and linoleum.

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Posterior producer and consumer risks are generally considered when there is prior information on the production process. Controlling the posterior risks allows the practitioners to assure, at the desired confidence levels, that the accepted and rejected lots are indeed acceptable and rejectable, respectively. In practice, the inclusion of previous data and subjective knowledge into an inferential process is frequently advantageous; see, e.g., Fernández [16,17], Han [18,19], Yan et al. [20], Ho and Huang [21], Jaheen and Okasha [22], Lee et al. [23], Lin et al. [24], Nyeo and Ansari [25] and Xu and Chen [26]. Nonetheless, the defect rate is assumed constant in traditional Poisson sampling inspection. This assumption is not realistic in many cases. Expert opinions and past data on the same or similar products are commonly available, and a probability density function can be used to summarize uncertainty about the defect rate.

Single sampling is the standard and simplest inspection procedure in industrial quality control. This conventional method was generalized in Govindaraju and Ganesalingam [27] by allowing resampling on nonaccepted lots. Recently, Wu et al. [28], Aslam et al. [29], Liu et al. [30] and Wu et al. [31] presented frequency-based acceptance sampling plans for resubmitted lots when the quality variable is normally distributed, and discussed the practical usefulness of resampling methods, whereas Fernández [32] proposed inspection schemes using previous defect count data and classical expected risks. In terms of inspection cost, resubmitted lot sampling is often superior to single sampling for examining high-quality products. This paper deals with the design of the resubmitted lot sampling plan based on prior information and Poisson defect counts with minimum sampling effort and controlled Bayesian posterior producer and consumer risks. A first sample is randomly selected from the submitted lot. If the number of nonconformities in the sample is sufficiently small, the lot is accepted. Otherwise, the lot can be resubmitted a determined number of times.

The remainder of this paper is organized as follows. Section 2 outlines resubmitted lot sampling inspection assuming that the number of nonconformities in a given product follows a Poisson distribution. Section 3 defines the posterior producer and consumer risks associated with a resubmitted lot defects-per-unit inspection scheme. An integer nonlinear program is formulated in Section 4 in order to find the best acceptance sampling plan. Next, Section 5 presents the computational methodology to determine the global solution of the constrained optimization problem. Several examples are included in Section 6. The prior uncertainty about the incoming defect rate per unit is described by a generalized truncated gamma distribution, which is the natural probability model when there exists expert opinions and/or historical data about similar products. Section 7 considers an application to paper manufacturing for illustrative purposes, whereas Section 8 offers some concluding remarks.

2. Resubmitted lot sampling inspection

Consider a manufacturing process in which the number of defects or nonconformities in a given product is a random variable N that follows a Poisson distribution with unknown defect rate per unit $\lambda > 0$. In such a case,

$$\Pr(N = i \mid \lambda) = \exp(-\lambda)\lambda^i/i!, \quad i = 0, 1, 2, \ldots.$$

A single sampling inspection scheme is described by the required sample size n and the rejection number r. A submitted batch is then accepted if the total number of nonconformities observed in a randomly chosen sample of n units is less than r; otherwise, it is rejected.

If resubmitted lot sampling is adopted, then the practitioner may sequentially select at most $k \geq 2$ random samples of size n from the lot. A sample of n units is first drawn at random to decide the acceptability of the lot. In case of nonacceptance, a maximum of $k-1$ random samples of size n can be selected from the lot. Thus, the original lot submission and at most $k-1$ lot resubmissions are allowed. Clearly, single sampling may be deemed as a particular case of resubmitted lot sampling when $k = 1$.

Assume for $i = 1, \ldots, k$ and $j = 1, \ldots, n$ that N_{ij} represents the number of nonconformities detected in the jth unit of the ith randomly selected sample from the submitted lot. In addition, suppose for $i = 1, \ldots, k$ that $T_i = N_{i1} + \cdots + N_{in}$ designates the total number of defects discovered in the ith inspected sample. The decision criterion associated with the resubmitted lot defects-per-unit inspection scheme (n, r, k) then asserts that the lot is accepted if and only if $T_i < r$ for some $i \in \{1, \ldots, k\}$. Taking into account that T_i follows the Poisson model with mean $n\lambda$, it is obtained that $\Pr(T_i \geq r \mid n\lambda) = G_r[n\lambda]$ for $\lambda > 0$ and $i = 1, \ldots, k$, where

$$G_r[x] = 1 - \exp(-x)\sum_{i=0}^{r-1}\frac{x^i}{i!}, \quad x > 0.$$

The probabilistic behavior of the sampling plan (n, r, k) is described by the so-called operating characteristic (OC) function, which is denoted by $F(\cdot\,; n, r, k)$. This function is defined for any defect rate $\lambda > 0$ as the corresponding probability of lot acceptance. Clearly, the OC function is decreasing, converges to 1 when $\lambda \to 0$, and tends to 0 as $\lambda \to \infty$. In view of the above-defined notation, it is deduced that the OC function can be expressed as

$$F(\lambda; n, r, k) = \sum_{i=0}^{k-1}\{G_r[n\lambda]\}^i\{1 - G_r[n\lambda]\}, \quad \lambda > 0,$$

which implies that

$$F(\lambda; n, r, k) = 1 - \{G_r[n\lambda]\}^k, \quad \lambda > 0.$$