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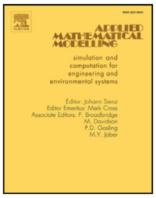

Reliability for discrete state systems with cyclic missions periods

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ABSTRACT

Different regimes, for example, different seasons, bull market/bear market, multiple missions etc., affect the performance of systems or products greatly in the real world. Thus, a dynamic reliability model, with cyclic multiple mission periods, is developed for non-repairable discrete state systems in this paper. By taking Laplace transforms and aggregated stochastic theory for describing the conditional probabilities of transitions among different mission periods and system states, we obtain closed form solutions of some reliability indexes and distributions of sojourn times with perfect and imperfect function. Three models of valve systems are considered in details based on different reliability structures, and a numerical example of marine reactor safety water injection system is also presented by applying our proposed methods for two special cases: (i) when the mission durations are fixed and; (ii) when the mission durations have exponential distributions.

1. Introduction

In many real applications, there are systems of valves operating under cyclic regimes. For instance, there are water storage dams, traffic lights control systems, sensor devices, and reactor safety injection systems. Water storage dam is used as draining off flood and irrigation, so the valves of the dam are needed to open and close cyclically according to mission requirements. Traffic lights transit among green, yellow and red with fixed and random mission durations. Sensor device is also needed to switch between listen and sleep back and forth, in order to monitor and save energy. Reactor safety injective system is composed of several kinds of valves and pumps, which are required to open and close to conduct coolant injection or not, based on reactor temperatures. Considering the costs of maintenance or the safety of crews, some valves are not repaired once they failed. For example, it does not matter that a few of light emitting diodes failed in traffic signal lights. From the point of safety and economy, valves or pumps in reactor safety injection system also can be regarded as non-repairable components. In this paper, just the aforementioned systems are what we are interested in.

The evaluation of system performance is a key issue for both producers and customers in mathematical modeling and project management, etc., along with many considerations as cost, time and quality. The accelerating deployment of large-scale web, Big data and complicated systems, which consist of the variety of dynamic attributes, have brought some new challenges in system performance measures. Performability, for example, acquires more attentions in the currently rapidly changing world, which was first introduced by John Meyer in 1980 in the context of the performance evaluation of aircraft

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control computers used by NASA as mentioned by Misra [1]. Now, numerous attributes of a system, like reliability, quality, availability and safety, etc., have been researched in the work of system modeling. Works on those can be found in much literature, e.g., in Refs. [2–12]. Especially, the emphasis on reliability can be seen in Refs. [2,5,7,8,10,12,13] for details. It is thus evident that the evaluation of system performance is a significant issue.

However, much of the literature does not take the influences of environments and multiple tasks on systems into account. As we know, different regimes influence the performance of systems or products greatly in the real world, for example, day/night, different seasons, bull market/bear market, multiple missions, etc.. Therefore, the factors of the environments or missions should be considered when evaluating the system performance. These regimes are often different from each other so that a system must be described by a dynamic model which involves different structure functions represented by components' indicator functions. Thus, there have been considerable interests in the applications of dynamic systems modifying their behaviors by several switching regimes, which are met in various application areas such as industry, medicine, traffic control, etc., especially in economics and finance. The history of the regime-switching models may be traced back to the early works of Quandt [14], Goldfeld and Quandt [15]. One of the main characters of these models is that system dynamic behaviors are allowed to be changed over time according to the states of an underlying Markov chain, which is also called a modulating Markov chain, see Ref. [16]. Switching of the active components (which corresponds to switching of the active system regimes) is often described via hidden Markov models (HMM) theory, for example see Ref. [17]. Hamilton [18] pioneered the use of the Markovian regime-switching models for fitting economic time series, and amount of research work is performed based on Markov regime-switching models by lots of scholars. Elliott and van der Hoek [19] gave an application of hidden Markov models to asset allocation problems. Nagy and Suzdaleva [20] proposed a recursive algorithm for estimation of mixtures with state-space components and a dynamic model of switching. The rainflow analysis in coastal engineering using switching second order Markov models was presented by Castillo et al. [21]. Pricing credit default swaps with bilateral counterparty risk in a reduced form model was provided by Liang et al. [22].

Though there are flourish achievements as mentioned in Refs. [14–23] in the regime-switching model, the existing papers mainly focus on the model's estimation algorithm, statistical perspective or applications in financial or other areas. In the system performance field, especially reliability, the Markov switching model has received less attention. To the best of our knowledge, Lim [24] considered a switching of the regimes (states) governing the lifetime distribution of the system and provided a Markov switching model describing this process. Ravishanker et al. [25] described a latent Markov process governing the parameters of a non-homogeneous Poisson process model, which characterizes the software development defect discovery process. A model about the evolution of system reliability performance under alternative environments was presented by Hawkes et al. [12], who gave the dynamic behaviors of a system represented by a finite-state Markov process operating under two alternating regimes. Note that performance under the first regime may affect future behavior under the second regime, thus the transition rate matrices under the two regimes will usually be different.

When performing various reliability tasks, non-repairable systems or products are treated differently from repairable systems or products. Examples of non-repairable systems are "one-shot" devices like light bulbs or more complex devices like pacemakers. In addition, valve systems in the industry engineering are often employed and it can be represented by a binary system, for instance, open and closed, good and bad, success and failure. From this view point, it is necessary to use a discrete system to describe the dynamic behaviors of systems. In general, a finite state Markov chain can be employed, for example, Fu and Koutras [26] and Cui et al. [27], which has been widely used in mathematical modeling, specially, in application fields such as reliability, finance and so on. When using the Markov chain technique, one-step probability transition matrix among its states must be established, and Laplace transforms are also used, for example, Colquhoun and Hawkes [28], Cui et al. [29] and Cui et al. [30]. All these techniques and factors will be considered and used in the present paper. Recently, some scholars also pay more attention to mission systems, non-repairable systems and multistage processes. For example, Wu and Hillston [31] obtained the mission reliability of semi-Markov systems under generalized operational time requirements with mission switching. Mo et al. [32] employed a method of multiple-valued decision diagram to analysis the efficient reliability of non-repairable phased-mission systems. Asadzadeh et al. [33] discussed the multistage process surveillance to improve the product reliability in manufacturing or service operations. Keeping above facts in view, the present paper deals with a dynamic reliability model with non-repairable discrete state systems under cyclic multiple-mission regimes. Some valve systems are used as examples to promote our thinking and motivations. A Markov regime-switching model and other stochastic processes theory are applied to describe the operating mechanism of discrete state systems. Besides, the distributions of some sojourn times under perfect and imperfect functioning are considered in the paper. The purpose of the paper is to develop some theoretical results that may be useful in modeling the evolution of system performance, for instance, quality and safety and so on, under multiple environments or missions. The authors believe that the model, and related results, can be used not only for the evaluation of system performance, but also in many contexts of project management, quality management, asset management, supply chain management and so forth.

The rest of the paper is organized as follows. In Section 2, the mission-switching model governed by a stochastic process and some assumptions are described. The reliability formulae and mean time to failures of the system are also presented. Based on the general results obtained in Section 2, three reliability systems are considered in details, their operating mechanisms and transition diagrams corresponding to Markov chains are investigated, respectively, in Section 3. Section 4 contains a detailed discussion on probability density functions of various sojourn times under system perfect and imperfect functioning. In Section 5, two special cases and some numerical examples are shown to illustrate the results obtained in the paper. Finally, the conclusions are given in Section 6.

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