Applied Mathematical Modelling 000 (2016) 1–9

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Short communication

Note on aggregating crisp values into intuitionistic fuzzy number

Jian Lin[a,b,*], Qiang Zhang[c]

[a] College of Computer and Information, Fujian Agriculture and Forestry University, Fuzhou 350002, China
[b] Smart Agriculture and Big Data Research Centre, Fujian Agriculture and Forestry University, Fuzhou 350002, China
[c] School of Management and Economics, Beijing Institute of Technology, Beijing 100081, China

ARTICLE INFO

Article history:
Received 3 July 2014
Revised 12 February 2016
Accepted 21 July 2016
Available online xxx

Keywords:
Group decision making
Intuitionistic fuzzy number
Aggregation
Facility location selection

ABSTRACT

Recently, a novel method for aggregating crisp values into intuitionistic fuzzy number is introduced by Yue. The aim of this short communication is to show how Yue's method can be improved to increase accuracy. A revised method is proposed to overcome the existing shortages. Some desirable properties of the revised method are studied. A problem of facility location selection is developed to illustrate the validity of proposed method.

1. Introduction

Due to the increasing complexity of real life decision making problems, aggregating a group's information and knowledge to make an optimal decision is an important research topic in group decision making. For the purpose of describing fuzzy information, the aggregated results of group decision information can be expressed by some special forms of fuzzy numbers, such as triangular fuzzy number, trapezoidal fuzzy number, intuitionistic fuzzy number and so on.

Since the intuitionistic fuzzy set [1] is more suitable for representing fuzzy information than classical fuzzy set introduced by Zadeh [2], numerous theories and methods are proposed to deal with group decision making problems under intuitionistic fuzzy setting. Due to the complex structure of intuitionistic fuzzy number, it is difficult to generate the intuitionistic fuzzy information in many practical group decision making problems. How to obtain and generate intuitionistic fuzzy information are seldom discussed. In order to develop an effective method for generating intuitionistic fuzzy information, Yue [3] proposed a novel method for aggregating crisp values into intuitionistic fuzzy number in group decision making. The right set and left set with respect to the elements of group attribute vector are introduced, and then the mean value of all elements in the right set and left set are calculated. Some linear transformations of these mean values and normalized process may generate the satisfactory degree, dissatisfactory degree and hesitation degree, respectively. However, the aggregating results derived by Yue's method are illogical in some cases and thus cause potential errors in group decision making. In addition, as everyone knows, some decision makers' opinion may have internal connection by the similar education background, political position, personal preference, etc. Nevertheless, the correlation among decision makers is not considered in Yue's method. As a useful tool for dealing with correlative problems in decision making, cooperative

* Corresponding author.
E-mail address: linjian36@163.com (J. Lin).

http://dx.doi.org/10.1016/j.apm.2016.07.020
0307-904X/

game has been widely studied. Shapley value is the most important solution concept for cooperative games, and is suitable to describe the correlative phenomenon among decision makers. The aim of this paper is to propose a revised method to aggregate crisp values into intuitionistic fuzzy number for group decision making. Based on the monotonicity of fuzzy measure, Shapley value can effectively reflect the correlation among decision makers. We exposed how to overcome the existing shortages by the revised method.

The rest of this paper is structured as follows. Section 2 introduces some basic concepts on intuitionistic fuzzy set. In Section 3, Yue's method is reviewed in detailed. Section 4 presents the revised method. Numerical example and comparison are shown in Section 5.

2. Preliminaries

In the following, some basic concepts on intuitionistic fuzzy set are introduced in detail.

Let X be a finite set. An intuitionistic fuzzy set [1] in X is expressed as:

$$\tilde{A} = \{< x, \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x) > | x \in X\},$$

where $\mu_{\tilde{A}} : X \to [0,1]$ is the membership function and $\nu_{\tilde{A}} : X \to [0,1]$ is the non-membership function, such that $\mu_{\tilde{A}}(x) + \nu_{\tilde{A}}(x) \le 1$ ($\mu_{\tilde{A}}(x)$ and $\nu_{\tilde{A}}(x)$ are the membership degree and non- membership degree of x to $\tilde{A}$, respectively). The hesitation degree of x to $\tilde{A}$ is denoted by $\pi_{\tilde{A}}(x) = 1 - \mu_{\tilde{A}}(x) - \nu_{\tilde{A}}(x)$. It is apparent that $\pi_{\tilde{A}}(x) \in [0,1]$.

For simplicity, $\tilde{\alpha} = (\mu_{\tilde{\alpha}}, \nu_{\tilde{\alpha}})$ is called an intuitionistic fuzzy number by Xu [4], such that $\mu_{\tilde{\alpha}} + \nu_{\tilde{\alpha}} \le 1$, $\pi_{\tilde{\alpha}} = 1 - \mu_{\tilde{\alpha}} - \nu_{\tilde{\alpha}}$ and $\mu_{\tilde{\alpha}}, \nu_{\tilde{\alpha}}, \pi_{\tilde{\alpha}} \in [0,1]$. Moreover, some operations and order relations to intuitionistic fuzzy numbers are introduced by Xu [4], shown as follows.

Definition 2.1. Let $\tilde{\alpha} = (\mu_{\tilde{\alpha}}, \nu_{\tilde{\alpha}})$ and $\tilde{\beta} = (\mu_{\tilde{\beta}}, \nu_{\tilde{\beta}})$ be two intuitionistic fuzzy numbers, then

(1) $\tilde{\alpha} \oplus \tilde{\beta} = (\mu_{\tilde{\alpha}} + \mu_{\tilde{\beta}} - \mu_{\tilde{\alpha}}\mu_{\tilde{\beta}}, \nu_{\tilde{\alpha}}\nu_{\tilde{\beta}})$,
(2) $a \otimes \tilde{\alpha} = (1 - (1 - \mu_{\tilde{\alpha}})^a, {\nu_{\tilde{\alpha}}}^a), a > 0$.

Definition 2.2. Let $\tilde{\alpha} = (\mu_{\tilde{\alpha}}, \nu_{\tilde{\alpha}})$ and $\tilde{\beta} = (\mu_{\tilde{\beta}}, \nu_{\tilde{\beta}})$ be two intuitionistic fuzzy numbers, then

(1) If $s(\tilde{\alpha}) > s(\tilde{\beta})$, then $\tilde{\alpha} \succ \tilde{\beta}$.
(2) If $s(\tilde{\alpha}) = s(\tilde{\beta})$, then
 (a) If $h(\tilde{\alpha}) = h(\tilde{\beta})$, then $\tilde{\alpha} = \tilde{\beta}$.
 (b) If $h(\tilde{\alpha}) > h(\tilde{\beta})$, then $\tilde{\alpha} \succ \tilde{\beta}$.

where $s(\tilde{\rho}) = \mu_{\tilde{\rho}} - \nu_{\tilde{\rho}}$ and $h(\tilde{\rho}) = \mu_{\tilde{\rho}} + \nu_{\tilde{\rho}}$ are the score function and accuracy function of intuitionistic fuzzy number $\tilde{\rho} = (\mu_{\tilde{\rho}}, \nu_{\tilde{\rho}})$, respectively.

3. Yue's method for group decision making

Due to the increasing complexity of modern society, many studies focus on the group decision making problems. Recently, Yue [3] developed a novel method to deal with group decision making problems. Let $A = \{A_1, A_2, \ldots, A_m\}$ be a finite set of alternatives, let $D = \{d_1, d_2, \ldots, d_t\}$ be the set of decision makers. $U = \{u_1, u_2, \ldots, u_n\}$ is the finite set of attributes, $W = (w_1, w_2, \ldots, w_n)$ is the weight vector of U. In Yue's method, decision makers evaluated the attributes by using crisp numbers. For each alternative, the expression of group decision making is denoted by a group decision matrix X_i ($i = 1, 2, \ldots, m$) as follows:

$$X_i = \begin{array}{c} \\ d_1 \\ d_2 \\ \vdots \\ d_t \end{array} \begin{array}{c} \begin{array}{cccc} u_1 & u_2 & \cdots & u_n \end{array} \\ \begin{pmatrix} x^i_{11} & x^i_{12} & \cdots & x^i_{1n} \\ x^i_{21} & x^i_{22} & \cdots & x^i_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ x^i_{t1} & x^i_{t2} & \cdots & x^i_{tn} \end{pmatrix} \end{array}$$

where x^i_{kj} is the value of attribute j evaluates by decision maker k corresponding to alternative i, $k = 1, 2, \ldots, t$, $j = 1, 2, \ldots, n$.

Let U_B and U_C be the set of benefit attributes and cost attributes, respectively. To measure all attributes in dimensionless, Yue [3] introduced a novel formula to normalize each attribute value in X_i, $i = 1, 2, \ldots, m$. The normalized group decision