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Drift reliability-based optimization method of frames subjected to stochastic earthquake ground motion

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ABSTRACT

The paper describes an inter-story drift reliability-based optimization method of frames subjected to stochastic earthquake loads. First, the formulas for eigenvalue and eigenvector, drift PSD functions, drift spectral moments, drift reliabilities, their first and second derivatives are derived based on random vibration theory. The computational procedure of drift reliabilities, their first and second derivatives are given in detail. Second, optimal problem of drift reliability-based optimization design is formulated in a dimensionless way. Optimal mathematic model is converted into unconstrained mathematic model using penalty function method. Gradient and Hessian matrix of penalty function are derived using drift reliabilities and structural mass, their first and second derivatives. Third, solution step of optimal problem is constructed using conjugate gradient method. Finally, optimization designs of two planar frames are demonstrated. Sensitivity analysis of optimum design indicates the drift reliability-based optimization method can obtain local optimum design.

1. Introduction

The optimization design of structures subjected to earthquake loads has been the subject of extensive research. Many researchers have addressed the optimization design methods for the seismic structures. These optimization methods can be classified into three categories based on the seismic response analysis methods. The first kind of optimization method for seismic structures is based on the static pushover analysis. For example, Xu et al. [1] develop a multicriteria optimization method for the performance-based seismic design of steel building frameworks under the equivalent static seismic loading. Ganzerli et al. [2] present a methodology for seismic design to minimize the structural cost with constraints on plastic rotations of the beams and columns. Liu et al. [3] describe a multiobjective optimization method for performance-based seismic design of steel moment frame structures. Chan et al. [4] present an optimization technique for the elastic and inelastic drift performance design of reinforced concrete buildings under response spectrum loading and pushover loading. Zou et al. [5] develop a computer-based technique that incorporates pushover analysis together with numerical optimization procedures to automate the pushover drift performance design of reinforced concrete buildings. The second kind of optimization method for seismic structures is based on time history response analysis. For example, Lagaros et al. [6] adopt an evolutionary algorithm to evaluate seismic design procedures for three-dimensional frame structures based on both linear and nonlinear time history analysis. Zou et al. [7] develop an optimal resizing technique for seismic drift design of concrete

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buildings subjected to response spectrum and time history loadings. Fragiadakis et al. [8] describe that a fully automated design methodology based on nonlinear response history analysis is proposed for the optimum seismic design of reinforced concrete structures. Liu et al. [9] develop an optimal method for seismic drift design of concrete buildings using the first and second derivatives of time history response with respect to design variables. The third kind of optimization method for seismic structures is based on random response analysis. For example, Papadrakakis et al. [10] describe a methodology for performing reliability-based structural optimum design of steel frames under seismic loading. The optimization part is realized with evolution strategies, while the reliability analysis is carried out with the Monte Carlo simulation method. Taflanidis et al. [11] develop a stochastic subset optimization method for reliability optimization and sensitivity analysis. The reliability analysis is also carried out using the Monte Carlo simulation method. The optimization method for seismic structures based on random response analysis is the most challenged among the three optimization methods for seismic structures.

Lateral drift is an important indicator which measures the level of damage to the structural and non-structural components of a building. Control of lateral drift is one of the key elements in the most seismic design codes [12–14]. In reality, the earthquake ground motion on structures is stochastic process in nature. The responses of a structure subjected to such uncertain loads are also stochastic in nature. Therefore, the aim of this paper is to develop an inter-story drift reliability-based optimization method of frame structures subjected to stochastic earthquake ground motion. The optimization method is formulated completely based on the random vibration theory. The paper is arranged as follows. In Section 2, the formulas for the eigenvalue and eigenvector, their first and second derivatives with respect to design variables are derived. In Section 3, the formulas for the inter-story drift PSD functions, their first and second derivatives are derived based on the pseudo excitation method. In Section 4, the formulas for the inter-story drift spectral moments, their first and second derivatives are given. In Section 5, the formulas for the reliabilities of the inter-story drifts, their first and second derivatives are derived based on the reliability formula in which the random process is assumed as two states Markov process. In Section 6, the detailed computational procedure of the reliabilities of the inter-story drifts, their first and second derivatives is given. In Section 7, the dimensionless formulas for the structural mass, its first and second derivatives are given. In Section 8, the optimal problem of the inter-story drift reliability-based optimization design is formulated. In Section 9, the dimensionless optimal mathematic model is formulated and converted into a sequence of appropriately formed unconstrained mathematic models using the interior point penalty function method. In Section 10, the gradient and Hessian matrix of the interior point penalty function are derived based on the information inter-story drift reliability and structural mass, their first and second derivatives. In Section 11, the solution step of the optimal design problem is given in detail based on the second order conjugate gradient method. In Section 12, the optimization designs of two planar frames are demonstrated using the optimization method proposed in this paper. The optimum designs, efficiency (compared with the optimization method proposed in reference [9]), other merits and limitations of the proposed method in this paper are discussed.

2. Eigenvalue and eigenvector, their first and second derivatives

The eigenvalue and eigenvector, their first and second derivatives are necessary to compute the PSD functions of inter-story drift, their first and second derivative. In this section, we will derive the formulas of the eigenvalue and eigenvector, their first and second derivatives.

When a structure with n degrees of freedom is subjected to the excitation of the ground horizontal acceleration time history $\ddot{x}_g(t)$, the governing equilibrium equation of structural dynamics can be expressed as follows:

$$\boldsymbol{M}\ddot{\boldsymbol{x}}(t) + \boldsymbol{C}\dot{\boldsymbol{x}}(t) + \boldsymbol{K}\boldsymbol{x}(t) = -\boldsymbol{M}\boldsymbol{E}_u\ddot{x}_g(t), \tag{1}$$

where $\boldsymbol{M}, \boldsymbol{C}, \boldsymbol{K}$ are the structural mass matrix, damping matrix and stiffness matrix, respectively. $\boldsymbol{x}(t), \dot{\boldsymbol{x}}(t)$ and $\ddot{\boldsymbol{x}}(t)$ are the displacement, velocity and acceleration vectors relative to the ground, respectively. For the planar frame structures, $\boldsymbol{E_u} = [1\,0\,0\,1\,0\,0 \cdots 1\,0\,0]^T$.

Rayleigh damping is used in this work, the structural damping matrix is

$$\boldsymbol{C} = \alpha_1\boldsymbol{M} + \alpha_2\boldsymbol{K}, \tag{2}$$

where

$$\alpha_1 = \frac{2\omega_1\omega_2(\zeta_1\omega_2 - \omega_1\zeta_2)}{\omega_2^2 - \omega_1^2}, \tag{3}$$

$$\alpha_2 = \frac{2(\zeta_2\omega_2 - \zeta_1\omega_1)}{\omega_2^2 - \omega_1^2}, \tag{4}$$

where ω_1 and ω_2 are the first and second natural frequency of the structure, respectively. ζ_1 and ζ_2 are the first and second mode damping ratios, respectively.

For the planar frame structures, the method for calculation of the structural mass matrix, damping matrix and stiffness matrix, their first and second derivative with respect to design variables is given in details in the literature [15].