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Backward bifurcation in a smoking cessation model with media campaigns

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ABSTRACT

Nowadays, most of the public policies to manage the social issues are focusing on motivating people via media campaigns. In view of this, we propose and analyze a nonlinear mathematical model to study the effect of media campaigns on smoking cessation. The equilibria of the model have been obtained and their stability discussed. Using center manifold reduction theory, we reduce the proposed model to a system of lower dimension. The reduced system contains all the necessary information regarding the asymptotic behavior of small solutions of the original system. The analysis shows that on changing one parameter of the system (reproduction number, $\mathcal{R}$, which depends on various other parameters), two different manifolds of fixed points cross each other and transcritical bifurcation occurs. Further, for large value of relapse rate the bifurcation is subcritical (backward). This shows that requirement $\mathcal{R} < 1$ is only necessary, but not sufficient, for smoking cessation. Numerical simulation also supports the analytically obtained results.

1. Introduction

Cigarette smoking and exposure to tobacco smoke is one of the biggest public health threats that kills more than 5 million people every year, which is more than the combined fatalities caused by HIV/AIDS, tuberculosis and malaria. Estimate reveals that if current trend continues, this figure may rise to over eight million people per year by 2030 [1]. There are more than one billion smokers worldwide out of which approximately 80% live in low- and middle- income countries [1]. About 30% of all cancer deaths, 80% of deaths due to chronic obstructive pulmonary disease and cardiovascular diseases are credited to the cigarette smoking [2]. It is the single most preventable cause of disease, disability, and death in many countries. Mostly youngsters are falling prey to the menace of smoking. Nowadays, cigarette smoking has become a common as well as fashionable trend among youngsters. Most of the youngsters smoke for the sake of their peer group or due to peer influence. The smoking rate among children with three or more friends who smoke is 10 times higher than the rate among those who report that none of their friends smoke [3]. People with smoking habit force their non-smoking peer to adopt this habit through coercion, teasing, bullying and rejection from the group [4].

Habit of cigarette smoking not only affects the smoker but also the society. An estimated 88 million non-smoking American residents, 54% of whom are children, are exposed to secondhand smoke [5]. This exposure causes serious diseases, such as heart disease and lung cancer in nonsmoking adults because they inhale almost the same poison in cigarette smoke as the actual smokers. Due to the devastating effect of smoking on society, it has become a major concern for government and

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health agencies. To monitor tobacco use among youth and design effectual control programs, the WHO and CDC have devised a Global Youth Tobacco Survey (GYTS), which tracks the tobacco consumption around the globe [6]. Bans and prohibitions are imposed on tobacco retailing to impede its consumption in various countries. Further the tobacco advertising, promotion, and sponsorship are also banned in 19 countries, which contain almost 6% of world's population [1]. The Government of India has also passed a law enforcing ban on smoking at public places, which took effect from October 2, 2008 [7,8].

The increasing focus of government and health care organizations on the prevention and control of addictive behaviors, such as tobacco and alcohol use have also reckoned interest of mathematical modelers in this area [9–15]. Smoking itself is not a disease but it attributes to many other health problems, like heart attacks, ischemic heart disease (stroke), chronic obstructive pulmonary disease, cerebrovascular disease, peripheral vascular disease, hypertension and cancer. Hence for controlling tobacco use, the preventive measures such as education and counseling are more effective than treatment [9]. Educating people regrading ill-effects of smoking is a rational strategy to lessen the prevalence of smoking and hence chances of developing lung cancer in the population [10]. The WHO Tobacco Free Initiative is raising awareness about global tobacco epidemic and preventive steps requisite to reduce tobacco use [16]. Anti-smoking media campaigns and advertising have reduced the prevalence of smoking among adults and adolescents significantly [17–19]. At present, there is a need of implementing country-wide mass media campaigns focusing on smoking cessation [20]. The media campaigns can influence peoples' behavior and lessen the magnitude of tobacco consumption by convincing smokers to quit smoking. A combination of large-scale media campaigns with state and community level strategies have the potential to curtail tobacco use across diverse populations [21]. Awareness programs by media and behavioral interventions have an enormous effect on the future course of an epidemic [22–26]. Therefore, to asses the impact of media campaigns on smoking cessation, we formulate and analyze a mathematical model for the spread of awareness amongst population in smoking age.

The aim of this paper is two fold. First is to present a mathematical model to study the impact of media on the smoking cessation so that effectual strategies can be devised. Second is, construction of a center manifold at a nonhyperbolic equilibrium point and determining the behavior of the flow on it. A *center manifold* at an equilibrium point is an invariant manifold tangent to the center subspace $\mathcal{E}^c$ of linearized system, i.e., the space spanned by the eigenvectors whose corresponding eigenvalues have zero real part, at that equilibrium point. This can be (locally) represented as the graph of a function $h : \mathcal{E}^c \to \mathcal{E}^s \oplus \mathcal{E}^u$, where $\mathcal{E}^s$ and $\mathcal{E}^u$ denote the stable and unstable subspaces, respectively. Here, h is a smooth function called the *reduction function* and satisfy conditions $h(0) = 0$ and $h'(0) = 0$, where 0 is the equilibrium point. The construction of center manifold is crucial to study the behavior of a dynamical system near a bifurcation point. At the bifurcation point the spectral bound (i.e., the dominant eigenvalue) of the linearized matrix of dynamical system near equilibrium point, crosses $Re(\lambda) = 0$. Since the dominant eigenvalue is crossing $Re(\lambda) = 0$, this implies that all the other eigenvalues are with negative real part. Then there exist a stable subspace $\mathcal{E}^s$ corresponding to eigenvalues with negative real part and a center subspace $\mathcal{E}^c$ corresponding to eigenvalues with zero real part. Therefore, it sufficient to study the behavior of flow on the center manifold to comprehend the dynamics of whole system. Since the dimension of center manifold is generally less than that of the original system, this technique is very useful for determining the local behavior of the complex dynamical systems. This technique provides an explicit expression for center manifold, yet its application to higher dimensional dynamical system is a deficient area and needs to be explored.

The rest of paper is organized as follows: In Section 2, a mathematical model to assess the impact of media campaigns on prevalence of smoking in the society is formulated. In Section 3, equilibria of the proposed model are obtained and their stability discussed. In Section 4, the explicit expression for the center manifold is obtained and thus the condition for backward bifurcation is worked out. The numerical simulation confirming the analytically obtained result is given in Section 5. Finally, the paper ends with a brief discussion.

2. Mathematical model

A lot of mathematical theory on the concept of diseases and epidemics is present in literature [27–30, and references therein]. The basic notion that underlies these models is that healthy people come in contact with infected individuals, contract infection and become infective. The dynamics of smoking is very similar to that of an epidemic. Here also the nonsmokers come in contact with smokers and start smoking due to peer influence. Based on epidemic models, we here propose a mathematical model to asses the impact of awareness created by media campaigns on the smoking cessation.

We consider a region with total population T at any time t. The whole population is divided into three classes: people who do not smoke cigarettes, Non-smokers (N); people who are regular smokers with non-zero FTND-score,[1] Smokers (S) and people who have quit smoking, Quitters (Q). The number of media campaigns to promote smoking cessation in the region under consideration is m. We assume that the mortality rate is balanced with the influx of people, so the total population is constant. It is considered that non-smokers start smoking due to peer influence and join the smoker class at a rate $\beta SN/T$. However, some smokers become aware of the ill-effects of smoking due to media campaigns, quit smoking and join the quitter class at a rate γSm. But the sudden withdrawal of nicotine supply from the body of an addicted person induces strong cravings, which are very hard to resist and some quitter relapse to smoking. Besides, the peer influence also tempts quitters to revert to the smoker class. In view of this, we have considered linear relapse σQ and non-linear relapse $\rho SQ/T$, both in the modeling process. The media

[1] Fagerstrom Test for Nicotine Dependence-score.

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