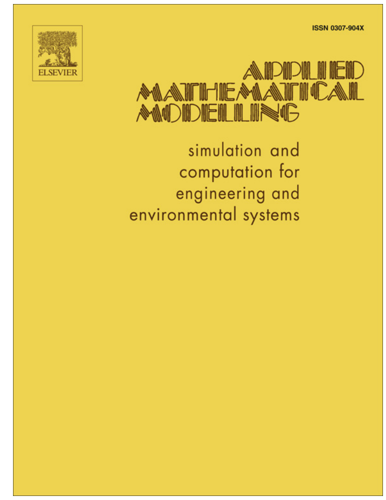

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A new analysis of stability and convergence for finite difference schemes solving the time fractional Fokker-Planck equation *

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Abstract

This paper presents a new analysis of stability and convergence for finite difference methods used to solve the time fractional Fokker-Planck equation. We show the monotone properties of the numerical solutions with respect to initial values and truncation errors, based on which we prove the stability and convergence under discrete L_1 norm.

2000 Mathematics subject classification: 65M12, 65M06, 35S10

Keywords: time fractional Fokker-Planck equations, finite difference methods

1 Introduction

This paper presents a new analysis of the stability and convergence for several finite difference methods used to solve the time fractional Fokker-Planck equation (FFPE)

$$\frac{\partial w}{\partial t} = {}_0D_t^{1-\alpha}\left[\frac{\partial}{\partial x}f(x) + K_\alpha\frac{\partial^2}{\partial x^2}\right]w(x,t), \quad a \le x \le b, 0 < t \le T \tag{1}$$

subject to the initial condition

$$w(x,0) = \varphi(x), \quad a \le x \le b$$

and boundary conditions

$$w(a,t) = g_1(t), \quad w(b,t) = g_2(t), \quad 0 < t \le T,$$

where the Riemann-Liouville fractional derivative of order $1-\alpha$ is

$${}_0D_t^{1-\alpha}w(x,t) = \frac{1}{\Gamma(\alpha)}\frac{\partial}{\partial t}\int_0^t \frac{w(x,s)}{(t-s)^{1-\alpha}}ds$$

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