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A new effective screening design for structural sensitivity analysis of failure probability with the epistemic uncertainty

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ABSTRACT

In this paper, two new sampling strategies are proposed to estimate the Morris' screening sensitivity measure and its improved version. The two new sampling strategies, which employ random sampling and quasi-random sampling respectively, compute the elementary effects of each factor at the same initial point and with a same step size in the input space. The new quasi-random sampling strategy performs better than the radial based sampling strategy and the new random sampling strategy performs almost the same with the radial based sampling strategy. Then, the improved version of the Morris' screening sensitivity measure is applied to estimate the effects of the epistemic uncertainty of random variables' distribution parameters on the failure probability using the new quasi-random sampling strategy. During this process, the principle of maximum entropy, fractional moments and dimension reduction method are used to estimate the failure probability with a good accuracy and a low computational demand. Several examples are employed to demonstrate the reasonability and the efficiency of the proposed strategy.

1. Introduction

In the field of engineering and science, uncertainties are often encountered, especially for structural reliability analysis. Generally, the uncertainties can be divided into two parts: aleatory uncertainty and epistemic uncertainty [1–3]. Aleatory uncertainty refers to the inherent variation of physical system and the environment (e.g. variation of the climate condition), and it is also known as irreducible, stochastic or objective uncertainty. Epistemic uncertainty, also known as reducible or subjective uncertainty, is usually due to the lack of knowledge of the physical system (e.g. lack of experimental data to describe the physical process accurately). The epistemic uncertainty can be reduced by collecting more experimental data or using more advanced scientific tools, while the aleatory uncertainty can be reduced. In structural reliability analysis, the uncertainty of basic design variables can be regarded as aleatory uncertainty since it is often due to the inherent variation of the physical structural system. This uncertainty is often described with probability theory [3–5], i.e. it is represented by probability density function (PDF). Usually, the distribution parameters of basic design variables are also uncertain due to the lack of experimental data or expert's guide, thus this uncertainty can be regarded as epistemic uncertainty. To represent the epistemic uncertainty, several theories can be used, such as probability theory [4,5], possibility theory [6,7], interval analysis [8], and fuzzy set theory [9]. Probability theory is widely used and well developed theory to represent uncertainty, as denoted by Helton and Oberkampf [3], historically, probability theory has provided the mathematical structure used to represent both aleatory uncertainty and epistemic uncertainty. For aleatory uncertainty, which is inherent and cannot be reduced, a unique PDF can be used to represent it. Since the epistemic uncertainty can be reduced, as knowledge of a practical structure or system is increased, different PDFs can be used to represent the epistemic uncertainty. However, for a given knowledge of a practical structure or system, a unique PDF is enough to represent the epistemic uncertainty. Usually, a limited knowledge of a practical problem is available, thus a unique PDF can be obtained based these limited knowledge to represent the epistemic uncertainty. Here, we will use the probability theory to describe the epistemic uncertainty since it is widely used in engineering application and well developed.

In structural reliability analysis, reliability sensitivity analysis is often used to rank the distribution parameters of basic design variables and guide the reliability based design [10]. In the traditional reliability sensitivity analysis, the distribution parameters of the basic design variables are specified as certain values. Thus, the reliability sensitivity often refers to the partial derivative of failure probability with respect to the distribution parameters at the

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nominal values. This sensitivity measure is just a local One At a Time (OAT) measure and cannot detect the interactions among factors [11]. In practical application, the distribution parameters are usually uncertain, and then the failure probability becomes a function of the distribution parameters. In this situation, the traditional local reliability sensitivity analysis is not suitable, and we need a global sensitivity measure to analyze the effects of the distribution parameters on the failure probability.

Contrary to local sensitivity analysis, global sensitivity analysis can evaluate the effect of one factor while the others are varying simultaneously and apportion the uncertainty of output to the uncertainty of the input factors. One of the most widely used global sensitivity method is the variance based method proposed by Sobol [12], Homma and Saltelli [13], Saltelli and Sobol [14], Iman and Hora [15]. Variance based sensitivity method is a model-free sensitivity measure based on the decomposition of variance, and can capture the influence of full range of variation of each factor. However, the drawback of the variance based method is the high computational cost, although several designs have been proposed to compute the sensitivity indices efficiently [16,17]. In the class of screening methods, the elementary effects method proposed by Morris [18] and improved by Campolongo et al. [19] is another good method for global sensitivity analysis. This method is a simple but effective approach to screen a few important input factors from many input factors in a model. The Morris method is based on the OAT method, which calculates the so-called elementary effect (EE). The EE is defined as the ratio between the change of the output and the change of the input factor, which is a local measure of sensitivity. Morris overcame this drawback by analyzing the distribution of EE, and uses the mean μ and standard deviation σ of this distribution as the sensitivity measure. Campolongo et al. [19] used the mean of the elementary effects in absolute value, denoted as μ^*, instead of the original μ to solve the type II errors [20] (failing to detect the factor with considerable influence on the model). Campolongo et al. [19] have shown experientially that μ^* is a good proxy of the variance based total sensitivity index S_T and Wainwright et al. [21] have shown the similarity between μ^* and S_T to confirm this conclusion. For the sensitivity analysis of large models, it is proven that μ^* is more effective than S_T [11,19]. For computational demanding numerical models, the surrogate models are often used to substitute the original models for global sensitivity analysis to further decease the computational cost, such as the polynomial chaos expansions [22–24], neural networks [25], state dependent parameter metamodelling [26,27].

Morris [18] suggested an efficient random sampling strategy via the trajectory based design to estimate the sensitivity measures. Since the sampling matrix is randomly generated, this strategy may lead to a non-optimal coverage of the input space, especially for models with large numbers of input factors. For this reason, Campolongo et al. [19] proposed an improved sampling strategy by maximizing the distance among the final selected trajectories. However, this improved strategy is unfeasible for large models due to the high computational cost to solve the combinatorial optimization problem. To solve this problem, Ruano et al. [28] suggested another improved sampling strategy based on trajectory design to decrease the computational cost for the optimization problem. As stated in [28], this procedure does not guarantee the final selected trajectories represent the global maximum distance among them, but these distances are at least locally maximized. Besides the trajectory based design, the radial based design [11,17] is also proposed as an efficient sampling strategy and Campolongo et al. [11] showed that the radial based design with the quasi random numbers performs better than the optimized trajectory based design. Since the EE is the ratio between the change of the output and the change of the input, the same initial point and same step size should be better than the above two strategy for computing EE. Thus, we propose a new sampling strategy called radial based design with the same step size to compute EE. We will test this new sampling strategy for estimating μ^* on a series of mathematical functions tested in [11].

In this paper, we will use the improved Morris' elementary effects method proposed by Campolongo et al. [19] to analyze the effects of the distribution parameters on the failure probability and the new radial based design with same step size is used to estimate the sensitivity measure. During the process of sensitivity analysis, the failure probability needs to be estimated at different sample points of distribution parameters, thus the Monte Carlo simulation (MCS) [29,30] or its improved versions, such as importance sampling [31,32], is unfeasible due to the high computational demand. The approximate approaches such as the first order reliability method (FORM) [33–36] and the second order reliability method (SORM) [34,37] need less computational cost but have relatively low accuracy for the complex models with high nonlinear performance function. If we can get the PDF f_G of the performance function, then we can easily get the failure probability by integrating f_G from $-\infty$ to 0 (the failure domain corresponds to the region where the performance function is less than 0). An effective method through combining the principle of maximum entropy, fractional moments and dimension reduction method is utilized by Zhang and Pandey [38] to effectively estimate the PDF of the performance function with much less samples than the MCS method. Here, we will use this method to estimate the PDF of the performance function and then calculate the failure probability at different sample points of the distribution parameters.

The rest of this paper is organized as follows. In Section 2, Morris's elementary effects method and the improved version by Campolongo et al. are introduced. At the same time, the trajectory based sampling strategy, radial based sampling strategy and the new radial based sampling strategy with same step size are introduced and compared by testing several mathematical functions. In Section 3, the principle of maximum entropy, fractional moments and dimension reduction method are introduced to estimate the failure probability. In Section 4, the improved elementary effects method proposed by Campolongo et al. is used to analyze the effect of distribution parameters on the failure probability. The method in Section 3 is used to estimate the failure probability and the new sampling strategy in Section 2 is used to estimate the sensitivity measure. Section 5 gives several examples to assess the effectiveness and efficiency of the proposed method. Finally, the conclusions are drawn in Section 6.

2. The elementary effects method and the sampling strategy

2.1. The Morris' elementary effect method and its improvement

The elementary effects (EE) method is originally proposed by Morris [18] to determine which input factors can be considered to have effects which are (1) negligible, (2) linear and additive, or (3) non-linear or involved in interactions with other factors. For the large and/or computational expensive models, researchers are more interested in the EE method than other quantitative techniques such as variance based method [12,13], since the EE method needs much less computational cost. Consider a model with k independent input factors $\mathbf{X} = (X_1, X_2, \ldots, X_k)$, which varies in a k-dimensional hypercube across p selected levels. Thus, the input space is discretized into a k-dimensional p-level grid Ω. For a given value of $\mathbf{X}$, the elementary effect of the ith input factors is defined as