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Modeling and analysis of a two-zooplankton one-phytoplankton system in the presence of toxicity

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ABSTRACT

In this study, we describe a two-zooplankton one-phytoplankton system that exhibits a Holling type II functional response in the presence of toxicity. Combined effort (E) is used to harvest the population. It is assumed that the phytoplankton is affected directly by an external toxic substance and the feeding of zooplankton on the affected phytoplankton is influenced indirectly by the toxic substance. All possible equilibria are determined for the system and the dynamical behavior of the system is investigated at each equilibrium point. The competitive exclusion principle is used to verify the coexistence of the zooplankton population. In order to incorporate the effects of the periodically varying environment, we consider the periodicity of the parameters and derive sufficient conditions for the uniformly strong persistence of the system. Combined effort, which is used to harvest the population, is treated as a control to develop a dynamic framework for investigating the optimal utilization of the resource, the sustainability properties of the stock, and the benefit earned from the resource, where Pontryagin's maximum principle is used to characterize the optimal control. The optimal system is derived and solved numerically using an iterative method with a Runge–Kutta fourth-order scheme. Our simulation results demonstrate that the optimal control scheme can obtain a sustainable ecosystem.

1. Introduction

In marine ecology, the term plankton is derived from the Greek adjective 'planktos,' which refers to errant elaborate drifting from one side to another on a water surface. Plankton flow with the ocean currents, but only a few are capable of independent movement and they can swim slowly with haphazard movements. Plankton refers to the collection of small microscopic organism that float or drift in great numbers in bodies of salt or fresh water, especially at or near the sea surface, where they provide the primary food source for fish and other larger organisms. Plankton are divided into three types: phytoplankton, zooplankton, and bacterioplankton. Phytoplankton are plant species that live near the water surface where there is sufficient light to support photosynthesis, such as Cyanophyta and Xanthophyta. The term zooplankton is derived from 'zoon' meaning animal and they comprise small protozoa or metazoans that feed on other plankton, as well as the eggs or larvae of larger animals. Finally, bacterioplankton are bacteria and Archaea that play important roles in remineralizing

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organic material in the water columns. Therefore, the plankton community can be classified into three groups, i.e., phytoplankton, zooplankton, and bacterioplankton, which act as producers, consumers, and recyclers, respectively.

Plankton inhabit the oceans, seas, lakes, and ponds, where their local abundance varies horizontally, vertically, and seasonally. This variability is mainly attributable to the availability of light. All plankton ecosystems are driven by solar energy, which confines primary production to the surface waters. A secondary variable is nutrient availability. Plankton are most abundant in surface waters and they live throughout the water column. At greater depths where no primary production occurs, zooplankton and bacterioplankton consume organometallic compounds that sink from the more productive surface waters above. In general, the growth of phytoplankton populations is dependent on the light levels and nutrient availability. However, the chief factor that limits growth varies among the regions of the world's oceans. Overall, the growth of phytoplankton in oligotrophic tropical and subtropical gyres is limited by the nutrient supply, whereas light usually limits phytoplankton growth in subarctic gyres. Changes in the stratification of the water column, the rate of temperature-dependent biological reactions, and the atmospheric supply of nutrients are also expected to have important effects on future phytoplankton productivity. Phytoplankton have a critical role in primary production, nutrient cycling, and food webs, and they comprise a significant proportion of the total production in aquatic systems. Indeed, almost 40% of the planet's total annual photosynthetic production is due to phytoplankton, even in salt marsh estuaries where the vascular plant biomass exceeds that of algae. Thus, the stocks of these microscopic planktonic algae play significant roles in marine reserves and fisheries management. Plankton form the basis of all aquatic food chains and phytoplankton occupy the first tropic level. Thus, phytoplankton support global biodiversity, i.e., by providing food for marine life and oxygen for animal life, while they also absorb half of the carbon dioxide, which may cause global warming. Phytoplankton provide food sources for numerous other organisms, especially zooplankton. Zooplankton grazing can significantly decrease the phytoplankton density. On average, an increase in the rate of zooplankton by 20% can decrease the phytoplankton population by approximately 75%.

In addition to the practical difficulties of data collection and interpretation for phytoplankton and zooplankton systems, oceanographers must address conceptual challenges when constructing models of these systems. Recently, several mathematical models have been generated of plankton systems [1–5]. For example, Pei et al. [6] considered the impact of harvesting on the coexistence and competitive exclusion of competitive predators. They proposed and investigated a two-zooplankton one-phytoplankton model with harvesting. Saha and Bandopadhaya [7] proposed a toxin-producing phytoplankton-zooplankton model where the toxin liberated by the phytoplankton species followed a discrete time variation. Gao et al. [8] considered the seasonality and periodicity of plankton dynamics. Rhodes et al. [9] considered an ecosystem model with a viral infection and demonstrated its effect as a regulator of the oceanic phytoplankton population. A simple nutrient–phytoplankton model was used to explore the dynamics of phytoplankton blooms by Huppert et al. [10].

Yunfei et al. [11] proposed a phytoplankton–zooplankton model with harvesting. They concluded that over-exploitation may cause the extinction of the population whereas appropriate harvesting should ensure the sustainability of the population. Tian et al. [12] investigated a reaction–diffusion system with nonlocal delay to describe the growth of two competing plankton types in an aquatic ecosystem. Zhang and Wang [13] considered a nutrient–phytoplankton–zooplankton model in an aquatic environment and they studied the global dynamics of the system. Rehim and Imran [14] considered a toxic phytoplankton–zooplankton system and analyzed the dynamical behavior of the system. Moreover, many studies have considered phytoplankton–zooplankton systems with a nutrient source, the coexistence of plankton, the toxic effect of the plankton system, or the effect of harvesting [15–22].

In the present study, we consider a one-phytoplankton and two-zooplankton population in the presence of toxicity. Our main objective is to study the effects of toxicity on the dynamics of the system. This study comprises three stages. First, we study the dynamical behavior of the system at all possible equilibria of the system. Second, we investigate the competitive exclusion principle to determine the coexistence of the zooplankton population, followed by the persistence of the system. Finally, the optimal control problem is formulated and solved using an iterative method with a Runge–Kutta fourth-order scheme. Numerical simulations are also performed to obtain the analytical results.

2. Model and its qualitative properties

We consider a one-phytoplankton two-zooplankton system with a Holling type II functional response. The biological relevance and ecological setup of the system are based on the following major assumptions.

It is assumed that x, y, and z are the sizes of the phytoplankton population and the first and second zooplankton populations, respectively, at time t.

In the absence of zooplankton, the growth of the phytoplankton population is logistic with an intrinsic growth rate r and carrying capacity K, or the maximum number of individuals that the environment can support.

It is considered that the phytoplankton population is consumed by two separate zooplankton populations and it is recycled into the zooplankton system. The zooplankton population consumes the phytoplankton populations with functional responses of the forms $\alpha xy/(a + x)$ and $\beta xz/(b + x)$, respectively, thereby contributing to their respective growth with $mxy/(a + x)$ and $nxz/(b + x)$.

It is assumed that the phytoplankton population is infected directly by an external toxic substance and the zooplankton population that feeds on this infected phytoplankton is affected indirectly by the toxic substance.